

# Contagion Behavior of High-frequency Trading in Future and Option Markets\*

PEILIN HSIEH<sup>†</sup>  
*Xiamen University*

YUQIANG GUO<sup>‡</sup>  
*Xiamen University*

HONGBIAO ZHAO<sup>§</sup>  
*Xiamen University*

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## Abstract

This study employs the classical self-exciting Hawkes point processes to model the dynamics and interaction of the order arrivals of the high-frequency trading S&P 500 Index futures and options in the U.S. market. We investigate the impact of a big overnight jump on the arrival times of orders by using the change of the branching ratios to measure whether there exist significant structure changes of contagion behavior after a big exogenous overnight jump occurs. We analyze all of the total orders, limit orders, market orders and cancelations in the ask and bid sides, and find that the order arrivals in both of the future and the option markets have significant structure changes after a big overnight jump. We also find that traders have different responses when facing negative-jump and positive-jump cases, and that there are little mutual-excitation evidence of the futures on the options in comparison to the strong self-excitation of the options itself.

**Keywords:** Contagion; Market Microstructure; High-frequency trading; Hawkes processes.

**JEL Classification:** C13 · C32 · C51 · C58

## 1 Introduction

Due to the availability of large high-frequency financial data, we can get more and more information about the investors' trading behavior, such as the dynamics of the order arrivals. For example, we plot the histogram of the future and call option limit order arrivals in Figure 1, the bottom figure of Figure 1(a) shows the call option limit order

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<sup>†</sup>The Wang Yanan Institute for Studies in Economics, Xiamen University, Xiamen, Fujian 361005, China. Email: peilin.h@gmail.com

<sup>‡</sup>The Wang Yanan Institute for Studies in Economics, Xiamen University, Xiamen, Fujian 361005, China. Email: guoyuqiang@xmu.edu.cn

<sup>§</sup>Department of Finance, School of Economics & Wang Yanan Institute for Studies in Economics, Xiamen University, Xiamen, Fujian 361005, China. Email: hongbiao.z@gmail.com

arrivals in the option market, and Figure 1(b) shows the future order arrivals. In fact, Figure 1 shows two common and interesting observations of the financial market. The first is that there exists order clustering, namely, orders do not arrive in evenly spaced intervals but usually arrive clustered in time, in one market; and the second is that there exists some joint clusterings between two different markets, i.e., the two markets may correspondingly have the order clusterings, when one market have more orders the other market also have more orders.

Explain the phenomena of order clustering and thus analyze the trading behavior of investors are very important, for example, for trading purposes it is very useful to be able to predict whether there is more buying or selling going on in the short term. However the most basic way to describe arrival of event counts is the basic Poisson Process, which can not depict the clusterings since the intensity of the Poisson Process  $\lambda$  is constant. The top figure of 1(a) shows the histogram of the simulated call option limit order arrivals which simulate from the univariate Hawkes Process. We can find the simulated histogram is in line with the histogram of real data, indicates that the order clustering may be illuminated by the Hawkes Processes. Hawkes processes, or also called self-exciting processes, are an extension of the basic Poisson process which aim to explain such clustering.

The Hawkes Processes (Hawkes, 1971a) were first used in seismology, modelling of earthquakes and volcanic eruptions, and then in various sciences such as ecology in wild-fire assessment (Peng et al., 2005), and even modelling of eruption of violence, e.g., on modelling civilian deaths in Iraq (Lewis et al., 2012), and on crime forecasting (Mohler et al., 2011). Since about 2005, Hawkes Processes have been widely applied in financial markets for modeling the contagion effects. It explains large number of works on trading activity and more generally interesting features of high-frequency econometrics as a modeling framework. For example, Aït-Sahalia et al. (2014) propose a mutually exciting jump process to capture the dynamics of asset returns, with periods of crises that are characterized by contagion. Bacry et al. (2013) reproduce microstructure noise (i.e. strong microscopic mean reversion at the level of seconds to a few minutes) and the typical Epps effect (Epps, 1979).<sup>1</sup> Also, many literature study the arrival times of orders in an order-book model in different markets by using self- or mutually exciting point processes. Large (2007) estimates the trading on an order book at London Stock Exchange

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<sup>1</sup>Chavez-Demoulin et al. (2005) and Chavez-Demoulin and McGill (2012) use Hawkes process to model of extreme returns in high-frequency financial time series. Grothe et al. (2014) propose a model with self-exciting point processes that can capture the typical features of multivariate extreme events observed in financial time series, namely, clustering behaviors in magnitudes and arrival times of multivariate extreme events, and time-varying dependence.

(LSE), using an appropriate parametric model which views the orders and cancelations as a ten-variate mutually-exciting Hawkes point process. Vinkovskaya (2014) focuses on the statistical modeling of the dynamics of limit-order books in electronic equity markets by using a multi-dimensional self-exciting point process.

Many documents demonstrate that jumps play an important role in explaining and forecasting asset prices in the financial markets using high-frequency data. Fleming and Paye (2011) use a sample of high-frequency returns for 20 heavily traded US stocks to show how microstructure noise distorts the standard deviation and kurtosis of returns normalized using realized variance. They conclude that jumps should be included in stock price models. Lee and Mykland (2012) propose a new empirical methods and find the evidence of jumps in underlying efficient price processes.

However, few works model the order arrivals across the markets of futures and options. Since high-frequency data are close to continuous-time observations, there are more information contained in high-frequency order-book data. Modeling the dynamics of order arrivals is of paramount importance for trading, risk management, derivative pricing, and to understand the behavior of investors. Moreover, there are few works that analyze the impact of jumps on the behavior of investors in the high-frequency future and option markets, while it is important to be investigated since the investors' trading decision reflect the systematic risk and asset valuation in the markets. Also, Easley et al. (1998) investigate the informational role of transactions volume in options markets, and find that option volumes contain information about future underlying prices. So, it is natural for us to wonder whether the option order arrivals would also contain information and to analyze traders' behavior further with the more detailed order arrivals.

As an attempt to address these issues, this study apply Hawkes processes to model the dynamics of limit-order books of Index future and option markets based on high-frequency data in the U.S. market. By defining an exogenous shock, we can then investigate how this exogenous shock can affect the order arrivals of Index futures and options. For the order classification, we follow Vinkovskaya (2014) to distinguish the limit-order book into six types of events: best limit-order bids, best limit-order asks, market buys, market sells, and best cancelations on the bid and ask sides. Since the market order and a cancelation can both decrease the quantities of outstanding shares in the order book, we put the market sells and best bid cancelations together, and the same to market buys and best ask cancelations, which are defined as the best MC orders. Combined with the total best ask and bid orders, we analyze six types of events: ask orders, limit asks, MC

asks, bid orders, limit bids and MC bids. After we empirically estimate the parameters, we can compare the different impacts (time scale and time decay) of each exogenous overnight jump between the call and put options, and compare different responses of the at-the-money (ATM) options, in-the-money (ITM) options and the out-of-the-money (OTM) options of each type of events. We also run significance test to check whether there is a significant structure change after the overnight jump.

By identifying the negative and positive overnight jumps, we analyze the dynamics of the order arrivals of the Index futures and options data for the day before, at and after the jump. We find that the order arrivals in both of the future and option markets have significant structure changes when there exist exogenous overnight price jumps. There exists a significant increment of the branching ratios, which introduced to measure the strength of contagion, after a negative jump, while a significant decrement in positive jump case in the futures market. In the option market, there are still significant increments of the branching ratios for most order types across different strikes when a negative jump occurs, while the result for the positive jump is not conclusive. Most of the branching ratios of the call options are larger than those of the put options given the negative jump occurrence while are less than those of the put options, especially for the ATM and ITM options, if the jump is positive. All the branching ratios of the ATM option are higher than those of the ITM's in both of the negative jump and positive jump cases. The branching ratios of the limit orders are larger than those of the MC orders for most of the call and put options at the negative jump. We also use the bivariate Hawkes processes to fit the futures and options simultaneously to check the mutual-excitation, and find little mutual-excitation evidence of the futures on the options but strong self-excitation inside the option market itself.

This study complements the literature on modeling the dynamics of orders by using the Hawkes processes. We model the arrivals times of orders of the Standard and Poor's 500 Index futures (SP) and future options (SPX) high-frequency data in an order-book model, and analyze the changes of investors' behavior when a sudden big price jump happens. Firstly, this study is the first work using the Hawkes model to study the trading patterns of order arrival in the options market. Bowsher (2007) was one of the first to apply the Hawkes processes to describe events in financial markets. He shows that there is a two-way interaction between trades and mid-quote changes, but only in the stock market and does not exploit the analytical properties of the model. Moreover, to the best of our knowledge, this study is the first to consider the interaction between Index futures and the future options by using a bivariate Hawkes model. Secondly, we consider the

influence of an exogenous overnight jump on the ordering behavior of traders. We introduce the branching ratio as a proxy to measure the contagion, i.e. the magnitude for one order to lead to another order, from Filimonov and Sornette (2012), while Filimonov and Sornette (2012) and Filimonov et al. (2014) apply the Hawkes process to estimate the percentage of price change caused by endogenous self-generated activity rather than the exogenous impact of news of novel information. There are other models that modeling contagion, however, the traditional approach of correlation models (Forbes and Rigobon, 2002) are hard to capture the non-linearity of dependency structure in the real world; and the copulas (Rodriguez, 2007) are not easy to model the dynamics and also get the associated parameters be estimated efficiently; also the classical model of autoregressive conditional duration (ACD) by Engle and Russell (1998) can be regarded as a simple special case of Hawkes Processes. Thirdly, We analyze all the information of the order flows: trade transactions, the limit orders, market orders, and cancelations. Cartea et al. (2014) apply the Hawkes process to model market order arrivals but does not consider the limit orders and cancelations. Hewlett (2006) uses a bivariate Hawkes process to model the EBS FX market, but only considers trade transactions. Large (2007) and Vinkovskaya (2014) consider limit orders, market orders, and cancelations on both the buy and the sell sides, but only consider the stock market and do not analyze how an exogenous jump could affect the order behavior of the investors.

The rest of the paper is constructed as follows. In Section 2, we briefly describe our model and present the maximum-likelihood function. Section 3 contains a description of the data, data classification and our methodology. Section 4 constitutes the main results, the significance tests, and the analysis about the results. Section 5 provides concluding remarks.

## 2 The Hawkes Processes

To model the dynamics and interaction of order arrivals in the high-frequency trading level, in this paper, we adopt a simple and widely-used special case of the classical Hawkes process (Hawkes, 1971a) – a linear self-exciting Hawkes process with an exponentially decaying kernel. We provide the basic mathematical outline of this process both for the univariate case and bivariate case. More general and mathematically regions definitions and distributional or statistical properties such as moments and maximum-likelihood function can be found in Hawkes (1971a), Hawkes (1971b), Ogata (1978), Ozaki (1979), Brémaud and Massoulié (1996), Embrechts et al. (2011) and Dassios and Zhao (2011).

## 2.1 Univariate case

In general literature, a *univariate linear Hawkes process of exponentially decaying kernel* is a point process  $N_t \equiv \{t_i\}_{i=0,1,2,\dots}$  with its conditional intensity function

$$\lambda_t = \mu(t) + \sum_{0 \leq t_i < t} \beta e^{-\delta(t-t_i)}, \quad (1)$$

where  $\mu(t) > 0$  is a time-deterministic function (*baseline or background intensity*);  $\beta > 0$  and  $\delta > 0$  are the influence size and exponential decay rate of the past events on the process, respectively. The non-negative  $\mathcal{F}_t$ -stochastic intensity  $\lambda_t$  conventionally satisfies

$$\Pr \{N_{t+\Delta t} - N_t = 1 \mid \mathcal{F}_t\} = \lambda_t \Delta t + o(\Delta t), \quad \Pr \{N_{t+\Delta t} - N_t > 1 \mid \mathcal{F}_t\} = o(\Delta t),$$

where  $\{\mathcal{F}_t\}_{t \geq 0}$  is a history of the process  $N_t$ , with respect to which  $\{\lambda_t\}_{t \geq 0}$  is adapted;  $\Delta t$  is a sufficiently small time interval and  $o(\Delta t)/\Delta t \rightarrow 0$  when  $\Delta t \rightarrow 0$ . It also satisfies the fundamental definition of intensity as the conditional mean arrival rate, i.e.

$$\lambda_t = \lim_{\Delta t \rightarrow 0} \frac{\mathbb{E}[N_{t+\Delta t} - N_t \mid \mathcal{F}_t]}{\Delta t}.$$

This point process  $N_t$  is counting the cumulative number of trade (or quote) arrivals within the time period  $[0, t]$ , and more importantly, its arrival intensity  $\lambda_t$  influenced by (conditional on) all of the past events occurring at  $\{t_i\}_{i=0,1,2,\dots}$  less than time  $t$ .

For model simplicity, we assume  $\mu(t)$  is fixed, i.e.  $\mu(t) \equiv \mu > 0$ . Thus the conditional intensity function are

$$\lambda_t = \mu + \sum_{0 \leq t_i < t} \beta e^{-\delta(t-t_i)},$$

Assuming stationarity, i.e.  $\frac{\beta}{\delta} < 1$ , we get the expected intensity as

$$\mathbb{E}[\lambda_\infty] = \frac{\mu}{1 - \frac{\beta}{\delta}}. \quad (2)$$

$\mu$  is the constant baseline intensity, which is the order number the traders would make in a unit time for the market without contagion.  $\beta$  is the constant size of self-excited jumps, measuring the instantaneous impact of each order. More specifically,  $\beta$  represents the size of another orders after a order made by the investors in the market.  $\delta$  is the constant rate of exponential decay, measuring the decay rate of the impact of each order afterward. Namely, the decay rate that the effect of a order to impact another orders would disappear after the order arrived. It's worthy to note that, if  $\beta \rightarrow 0$  or  $\delta \rightarrow \infty$ , then  $\lambda_t = \mu$ , it becomes to a Poisson process.

Our study follow the Embrechts et al. (2011), introduce the intensity function as an-

other form:

$$\lambda_t = \mu + \theta \sum_{0 \leq t_i < t} \delta e^{-\delta(t-t_i)}, \quad (3)$$

The important quantity *branching ratio*  $\theta = \frac{\beta}{\delta}$  which measures the strength of contagion, i.e. the magnitude for one order to lead to another order (Filimonov and Sornette, 2012).

Given the observation of  $\{t_i\}_{i=0,1,2,\dots,N}$ , the log-likelihood function of the univariate Hawkes process can be derived from Ozaki (1979), i.e.

$$\ln \mathcal{L}(\{t_1, \dots, t_N\}) = -\mu t_N - \theta \sum_{i=1}^N (1 - e^{-\delta(t_N - t_i)}) + \sum_{i=1}^N \ln[\mu + \theta \delta A(i)], \quad (4)$$

where

$$A(i) = \begin{cases} \sum_{t_j < t_i} e^{-\delta(t_i - t_j)}, & \text{for } i \geq 2, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

## 2.2 Bivariate case

The univariate Hawkes process only captures the self-excitation of the events, whereas in the bivariate setting, in addition to self-excitation, there is the possibility of mutual-excitation (or cross-excitation), for which jumps of one process can elevate the intensity of the other process and hence induce more jumps afterward for the other process. Many previous studies have applied the bivariate (multivariate) point processes to model the cross-excitation effect of the financial market. Engle and Lunde (2003) specify a model for a bivariate sequence of durations to jointly analyze the trade and quote arrivals. Bowsher (2007) also uses bivariate point process model of the timing of trades and mid-quote changes and relates to the market microstructure literature, but implies a conditional intensity in continuous time for midquote change events. Embrechts et al. (2011) show that the multivariate Hawkes processes offer a versatile class of point processes capable of modeling extremal behavior of financial time series, such as the positive and negative jump cases.

The intensity of a linear bivariate Hawkes process with additional the cross-excitation can be expressed by

$$\lambda_m(t) = \mu_m + \sum_{n=1}^2 \sum_{t_{n,i} < t} \beta_{mn} e^{-\delta_{mn}(t-t_{n,i})}, \quad m = 1, 2, \quad (6)$$

namely,

$$\begin{cases} \lambda_1(t) = \mu_1 + \sum_{t_{1,i} < t} \beta_{11} e^{-\delta_{11}(t-t_{1,i})} + \sum_{t_{2,j} < t} \beta_{12} e^{-\delta_{12}(t-t_{2,j})}, \\ \lambda_2(t) = \mu_2 + \sum_{t_{1,i} < t} \beta_{21} e^{-\delta_{21}(t-t_{1,i})} + \sum_{t_{2,j} < t} \beta_{22} e^{-\delta_{22}(t-t_{2,j})}. \end{cases}$$

In this study, we set  $\delta_{11} = \delta_{12} = \delta_1$  and  $\delta_{21} = \delta_{22} = \delta_2$  as in Dassios and Zhao (2013).  $\beta_{11}, \beta_{22}$  are the sizes of self-excited jumps, capturing the instantaneous impact of self-contagion within the same market; and  $\beta_{12}, \beta_{21}$  are the sizes of cross(mutual)-excited jumps, capturing the instantaneous impact of cross(mutual)-contagion across both markets.

Following Embrechts et al. (2011), in this study, we use

$$\begin{cases} \lambda_1(t) = \mu_1 + \theta_{11} \sum_{t_{1,i} < t} \delta_1 e^{-\delta_1(t-t_{1,i})} + \theta_{12} \sum_{t_{2,j} < t} \delta_1 e^{-\delta_1(t-t_{2,j})}, \\ \lambda_2(t) = \mu_2 + \theta_{21} \sum_{t_{1,i} < t} \delta_2 e^{-\delta_2(t-t_{1,i})} + \theta_{22} \sum_{t_{2,j} < t} \delta_2 e^{-\delta_2(t-t_{2,j})}. \end{cases} \quad (7)$$

where  $\theta_{11} = \frac{\beta_{11}}{\delta_1}$ ,  $\theta_{12} = \frac{\beta_{12}}{\delta_1}$ ,  $\theta_{21} = \frac{\beta_{21}}{\delta_2}$  and  $\theta_{22} = \frac{\beta_{22}}{\delta_2}$ .  $\theta_{11}$  and  $\theta_{22}$  represents for the self-contagion effect that the magnitude for one order to lead to another order within the same market,  $\theta_{12}$  and  $\theta_{21}$  represents for the cross (mutual)-contagion effect across both markets, which the magnitude for one order in one market to lead to another order in another market.

The stationarity condition for this bivariate case is

$$\frac{1}{2} \left[ \theta_{11} + \theta_{22} + \sqrt{(\theta_{11} - \theta_{22})^2 + 4\theta_{12}\theta_{21}} \right] < 1. \quad (8)$$

The log-likelihood function for the bivariate process can be written as

$$\ln \mathcal{L} (\{t_{m,i}\}_{i=1,\dots,N}^{m=1,2}) = \sum_{m=1}^2 \ln \mathcal{L}_m, \quad (9)$$

where

$$\ln \mathcal{L}_m = -\mu_m t_{m,N} - \sum_{i=1}^N \sum_{n=1}^2 \theta_{mn} (1 - e^{-\delta_m(t_{n,N} - t_{n,i})}) + \sum_{t_{m,i} < t_{m,N}} \ln \left[ \mu_m + \sum_{n=1}^2 \theta_{mn} \delta_m R_{mn}(i) \right], \quad (10)$$

and

$$R_{mn}(i) = \begin{cases} e^{-\delta_m(t_{m,i} - t_{m,i-1})} R_{mn}(i-1) + \sum_{t_{m,i-1} \leq t_{n,j} < t_{m,i}} e^{-\delta_m(t_{m,i} - t_{n,j})}, & \text{if } m \neq n, \\ e^{-\delta_m(t_{m,i} - t_{m,i-1})} [1 + R_{mn}(i-1)], & \text{if } m = n. \end{cases} \quad (11)$$

and  $R_{mn}(0) = 0$ .

Namely,

$$\begin{aligned}
\ln \mathcal{L}_1 &= -\mu_1 t_{1,N} - \sum_{i=1}^N \theta_{11} (1 - e^{-\delta_1(t_{1,N}-t_{1,i})}) - \sum_{j=1}^N \theta_{12} (1 - e^{-\delta_1(t_{2,N}-t_{2,j})}) \\
&\quad + \sum_{t_{1,i} < t_{1,N}} \ln[\mu_1 + \theta_{11}\delta_1 R_{11}(i) + \theta_{12}\delta_1 R_{12}(i)], \\
\ln \mathcal{L}_2 &= -\mu_2 t_{2,N} - \sum_{i=1}^N \theta_{21} (1 - e^{-\delta_2(t_{1,N}-t_{1,i})}) - \sum_{j=1}^N \theta_{22} (1 - e^{-\delta_2(t_{2,N}-t_{2,j})}) \\
&\quad + \sum_{t_{2,j} < t_{2,N}} \ln[\mu_2 + \theta_{21}\delta_2 R_{21}(j) + \theta_{22}\delta_2 R_{22}(j)].
\end{aligned}$$

where

$$\begin{cases} R_{11}(i) = e^{-\delta_1(t_{1,i}-t_{1,i-1})} [1 + R_{11}(i-1)], \\ R_{12}(i) = e^{-\delta_1(t_{1,i}-t_{1,i-1})} R_{12}(i-1) + \sum_{t_{1,i-1} \leq t_{2,j} < t_{1,i}} e^{-\delta_1(t_{1,i}-t_{2,j})}, \\ \\ R_{22}(j) = e^{-\delta_2(t_{2,j}-t_{2,j-1})} [1 + R_{22}(j-1)], \\ R_{21}(j) = e^{-\delta_2(t_{2,j}-t_{2,j-1})} R_{21}(j-1) + \sum_{t_{2,j-1} \leq t_{1,i} < t_{2,j}} e^{-\delta_2(t_{2,j}-t_{1,i})}. \end{cases}$$

With the log-likelihood functions, we can estimate the Hawkes processes via the maximum likelihood estimation (MLE) for the arrival orders in an order-book model.

### 3 Data and methodologies

We begin this section by a brief discussion of the data source, and then classify the options data into different order types: best limit-order asks, best limit-order bids, market buys, market sells, and the best cancelation of ask and bid sides. Since the market order and a cancelation can both decrease the quantities of best quotes, we put the market sells and best bid cancelations together as best MC asks and count market buys and best ask cancelations as best MC bids. So we divide the order book data into four types: limit asks, limit bids, MC asks, and MC bids. At last, we describe the methodology that classifies sub-samples of our interest, as the order arrival times of sub-samples are used for estimating our model.

#### 3.1 The Data

Our study is based on the Standard and Poor's 500 Index futures (SP) and future options (SPX) high-frequency data, which was provided by Thomson Reuters, and both are traded in Chicago Board Options Exchange (CBOE). The database consists of high-frequency trades (transactions) and quotes (order arrivals) for all the Index futures and options. As S&P 500 futures and options are the contracts among the most actively traded index products on CBOE, it is common to observe several orders occurring within one

second.

We extract futures data from the Trades table and option data from the Quotes table. The sample period covers two years from January 1, 2010 to December 31, 2011 (with total 517 days: 259 days for 2010 and 258 days for 2011). Both datasets contain closing price, expiration date, total volume, and open interest. The strike price, and best bid and best prices are only available for options contracts. Therefore, this paper focuses on analysis of arrivals of trading for futures and arrivals of quotes for options. We apply the following rule to filter unnecessary data. First, market events (i.e. trades or quotes) occurring outside of normal trading day hours (9:30 AM to 4:15 PM) are deleted from the datasets, because the level of activity is low outside the trading hours, and the dynamics of the limit-order book are entirely different from those that are typically observed during the trading day. Second, we target at the most intensively traded option contracts which generally are nearest term with time to maturity less than one month. Third, we delete the data which has invalid records: quote prices that are \$0.00 or greater than \$999.99 and unusually large quote sizes.

### 3.2 Data Classification

For options, the Quotes table can tell us about the decreases and increases in the quantities of outstanding shares at the best-bid and best-offer prices. An increase always corresponds to a limit order while both market order and a cancelation can lead to a decrease. Following Vinkovskaya (2014), out of the six limit-order book events: limit bids, limit asks, market buys, market sells, cancelations at the bid and cancelations at the ask prices, we classify the information in the Quotes table into the following four events:

- (1) limit bids,
- (2) limit asks,
- (3) MC asks (market sells and bid cancelations combined),
- (4) MC bids (market buys and ask cancelations combined).

The algorithm for classifying each events is as follows. For each row in the Quotes table we compare the bid price  $p_t^b$  to the bid price in the previous row  $p_{t-1}^b$  along with the bid sizes at those levels,  $Q_t^b$  and  $Q_{t-1}^b$ .

- If  $p_t^b > p_{t-1}^b$ , then we classify it as a limit-bid order.
- If  $p_t^b < p_{t-1}^b$ , then we classify it as a MC-bid order.
- If  $p_t^b = p_{t-1}^b$ :

- If  $Q_t^b > Q_{t-1}^b$ , then we classify it as a limit-bid order.
- If  $Q_t^b < Q_{t-1}^b$ , then we classify it as a MC-bid order.
- If  $Q_t^b = Q_{t-1}^b$ , then no classification.

Similarly, we compare the ask price  $p_t^a$  to the ask price in the previous row  $p_{t-1}^a$  along with the ask sizes at those levels,  $Q_t^a$  and  $Q_{t-1}^a$  to classify the ask orders.

- If  $p_t^a > p_{t-1}^a$ , then we classify it as a MC-ask order.
- If  $p_t^a < p_{t-1}^a$ , then we classify it as a limit-ask order.
- If  $p_t^a = p_{t-1}^a$ :
  - If  $Q_t^a > Q_{t-1}^a$ , then we classify it as a limit-ask order.
  - If  $Q_t^a < Q_{t-1}^a$ , then we classify it as a MC-ask order.
  - If  $Q_t^a = Q_{t-1}^a$ , then no classification.

Thus we can select the order arrivals data of each of the four types respectively.

Figure 2 shows an example of one day for the histogram of the futures and option limit orders that extracted by the classification algorithm as above. Figure 2(a) shows the histogram for the day of May 24, 2010, Figure 2(b) shows the histogram for the day of May 25, 2010, and Figure 2(c) shows the histogram for the day of May 26, 2010. From these figures, we can find that the behavior of histogram for futures, call option asks, call option bids, put option asks, and put option bids are different and that the behavior for each day are also different.

\* (Figure 2 here)

### 3.3 Methodologies

In this study, we investigate how a big overnight price jump (which is exogenous) can affect the dynamics of the order arrivals of Index futures and options. We define an large overnight jump if the absolute value of overnight return of the S&P Index futures is large than 2%. By calculating the Index future overnight return through all data sample, we find that there exist 11 overnight jumps in the database. Table 1 shows the descriptive information of the 11 days which an overnight jump happens, which means that there are an average of 5–6 days per year in our database indicates that the large overnight jump we define is a rare event . We choose one negative jump and one positive jump respectively as two representative cases to analyze the changes of investor behavior through the order arrivals after a price jump happened. The negative jump happens at May 25, 2010, and the positive jump happens at October 27, 2011, which is defined as the jump

day. Then we extract the data of the jump day, one day before the jump day, and one day after the jump day, respectively, namely, we extract the data of May 24, 2010, May 25, 2010, May 26, 2010, October 26, 2011, October 27, 2011, and October 28, 2011 of Index futures and options.

\* (Table 1 here)

For the option data, we use the following method to filter the at-the-money (ATM) options, in-the-money (ITM) options, and out-of-the-money (OTM) options, respectively. Firstly, we find the minimum price  $P_{min}$  and the maximum price  $P_{max}$  of the underlying Index futures of each day, and then consider all the options which have strike price less than  $K_{max} = (1 + 1\%) \times P_{max}$  and greater than  $K_{min} = (1 - 1\%) \times P_{min}$  as the ATM options. Thus we can construct an interval  $[K_{min}, K_{max}]$  that all the options that have strike price belongs to this interval are classified as ATM options. Secondly, we can get four strike price  $[K_1^A, K_2^A, K_3^A, K_4^A]$  that evenly divide the ATM interval. Then we extract all the ATM call option and ATM put option data that have strike price  $K_1^A, K_2^A, K_3^A,$  and  $K_4^A,$  respectively. Thirdly, according to the evenly distance that divides the ATM interval, we can correspondingly get the ITM  $[K_1^I, K_2^I, K_3^I, K_4^I]$  and OTM interval  $[K_1^O, K_2^O, K_3^O, K_4^O]$ .<sup>2</sup> It is noteworthy that the ITM interval for call options is the OTM interval for put options, and the OTM interval for call options is the ITM interval for put options. So we can get all the data of ATM options, ITM options and OTM options, respectively.

From subsection 3.2, we can classify each extracted strike price of options into four categories of orders. The ask orders are the orders that includes both the limit-ask orders and the MC-ask orders. The same definition for the bid orders. Combined with the total ask and bid orders, we can analyze six types of events: ask orders, limit asks, MC asks, bid orders, limit bids, MC bids.

## 4 Results

In this section, we discuss the main results: Section 4.1 shows the results of the univariate case; Section 4.2 presents the significance test results, in Section 4.3 we estimate the bivariate models of the Index futures and the options.

### 4.1 Univariate Case

We use the univariate Hawkes process to model both the order arrivals of the S&P Index futures and options.

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<sup>2</sup>The superscripts A, I, and O for the strike price are represent for ATM option, ITM option, and OTM option respectively.

#### 4.1.1 The Future Data

We use the univariate Hawkes process to fit all the extracted future data of one day before the jump, the jump day, and one day after the jump, respectively. The day of May 24, 2010, May 25, 2010 and May 26, 2010 are three days around a negative overnight jump, and the day of October 26, 2011, October 27, 2011 and October 28, 2011 are three days that around a positive overnight jump.

The first row of the Panel A of Table 2 shows the estimate results of the branching ratio for the data of the negative jump case, while the second row of Panel A shows the results of the positive jump case. We find that the branching ratio increases at the day that jump happens and then decreases one day after the jump day for the negative jump case, while decreases all of the three days for the positive jump case.

\* (Table 2 here)

#### 4.1.2 The Option Data

We show the results for the case of negative overnight jump and the case of positive overnight jump respectively.

**Negative Overnight Jump Case** We use the univariate Hawkes process to fit all the extracted option data of one day before the negative jump, the jump day, and one day after the jump, respectively. Table 3 shows the number of each type of events: ask orders, limit asks, MC asks, bid orders, limit bids, and MC bids, of each type of options: the ATM calls, ITM calls, OTM calls, ATM puts, ITM puts, and OTM puts for the three days. We find that:

- (1) The number of put option orders are larger than the number of call option orders, especially after the jump happens, which implies that the investors prefer put options to the call options;
- (2) For the call and most put options, the traders quote more intensively for the ITM options, and then for the ATM options, and the last for the OTM options. While for the strike price of  $K_1$  and  $K_2$ , the traders choose the ITM options at first and the ATM options at last after the jump day;
- (3) For the call options, the traders decrease their order arrivals when a jump happens no matter for the ATM, ITM, and OTM options, while for the put options the traders have a most high frequency at the ATM asks and other cases are the same to call options;

(4) All the limit orders have less arrivals than the MC orders, which implies that the option investors are prefer to make MC orders than the limit orders.

\* (Table 3 here)

Table 4 shows all the estimate results of branching ratio for the different type of the three days data. We have several interesting findings from the Table 4. First, the branching ratio  $\theta = \frac{\beta}{\gamma}$  increases at the jump day for all the ATM, ITM, and OTM call and put options, and decreases at one day after the jump day for all the ITM and OTM options, namely,  $\theta_B < \theta_T$  for all the options, and  $\theta_T > \theta_A$  for all the ITM and OTM options.<sup>3</sup> We know that the branching ratio represents for the magnitude that an order can lead to another order, which implies that an order may lead to more orders when a sudden jump happens and would decrease towards the magnitude before the jumps later. Second, when comparing the call and put options, we can find that most of the branching ratio of the call options are larger than those of the put options,  $\theta_C > \theta_P$ .<sup>4</sup> Combined with the number of arrival orders for call options are less than the put options especially after the jump happens, we can conclude that the investors maybe more concern about the call options and active more frequently for the call options when a jump happens than for the put options. Third, by comparing all the ATM options and the ITM options, we find that all the branching ratios of the ATM option are higher than those of the ITM's, i.e.,  $\theta_A > \theta_I$ . Fourth, the branching ratio of the ATM options are less than those of the OTM's, i.e.,  $\theta_A < \theta_O$ , for all the data in one day before the jump day and the jump day, which are contrary to the case of one day after the jump day.<sup>5</sup> Fifth, for the ATM type of Call options, we find that the branching ratio of all the ask orders, which include the ask orders, the limit asks, and the MC asks, is less than the branching ratio of all the bid orders, which include the bid orders, the limit bids, and the MC bids, for one day before and one day after the jump day, while large than the branching ratio of the bid orders for the jump day, respectively. For both of the ITM and OTM put options, the branching ratio of all the ask orders are less than those all of the bid orders for the data of one day after the jump day. All other cases show contrary results that  $\theta_A > \theta_B$ ,  $\theta_{AI} > \theta_{BI}$ , or  $\theta_{Am} > \theta_{Bm}$ .<sup>6</sup> Sixth, by comparing the limit asks with the MC asks and the limit bids with MC bids, we find that the branching ratio of the limit orders are large than those of the MC orders for most of the call and put options, i.e.,  $\theta_l > \theta_m$ , which is consistent with the findings that all the limit orders have less arrivals than the MC orders from Table 2, implies that the investor are more concerned about the limit orders.

<sup>3</sup> $\theta_B$ ,  $\theta_T$ , and  $\theta_A$  are represents for the branching ratio of one day before the jump day, the jump day, and one day after the jump day respectively.

<sup>4</sup> $\theta_C$  and  $\theta_P$  are represents for the branching ratio of the call option and the put option respectively.

<sup>5</sup> $\theta_A$ ,  $\theta_I$ , and  $\theta_O$  are represents for the branching ratio of the ATM option, the ITM option, and the OTM option respectively.

<sup>6</sup> $\theta_A$ ,  $\theta_B$ ,  $\theta_{AI}$ ,  $\theta_{BI}$ ,  $\theta_{Am}$ , and  $\theta_{Bm}$  are represents for the branching ratio of the ask orders, the bid orders, the limit asks, the limit bids, the MC asks, and the MC bids respectively.

\* (Table 4 here)

**Positive Overnight Jump Case** We use the univariate Hawkes process to fit all the extracted option data of one day before the positive jump, the jump day, and one day after the jump, respectively. Table 5 shows the number of each type of events: ask orders, limit asks, MC asks, bid orders, limit bids, and MC bids, of each type of options: the ATM calls, ITM calls, OTM calls, ATM puts, ITM puts, and OTM puts for the three days. From Table 5 we find that: (1) Different from the negative jump case, the number of put option orders are larger than the number of call option orders only for most of the OTM options, which is contrary to the situation of the ITM options and most of the ask orders of the ATM options. (2) For the call options, the investors are the best prefer to the ITM options, and then the ATM options, and the last prefer to the OTM options. While for the put options data of one day before the jump happens, the investors choose the ITM options at first and the ATM options at last, and the investors have no conclusive preference for the other two days; (3) For the call options, the investors decrease their order arrivals when a jump happens for most of the ATM, ITM, and OTM options, while for the put options the investors have a most high frequency at the ATM options and other cases are basically the same to call options; (4) Most of the limit orders have larger arrivals than the MC orders, implies that the option investors are prefer to make limit orders than the MC orders when facing with a positive jump, which is contrary to the negative jump case. All the findings show that there are less consistent performance when a positive jump happens than that for the negative jump case.

\* (Table 5 here)

Table 6 shows the estimate results of branching ratio for the different type of the three days data.<sup>7</sup> Like the negative jump case, we also have several findings from Table 6. First, the branching ratio  $\theta = \frac{\beta}{\delta}$  decreases at the jump day only for the ATM, call and put options, and increases at one day after the jump day, namely,  $\theta_B > \theta_T$  first and then have  $\theta_T < \theta_A$  for all the ATM options. For the OTM call options, we have the results just like the negative jump case that the branching ratio increases at the jump day, and decreases at one day after the jump day. Second, by comparing the call and put options, we find that most of the branching ratios of the call options are less than those of the put options,  $\theta_C < \theta_P$ , especially for the ATM and ITM options. Combined with the findings that for most of the ATM and ITM options, the number of arrival orders for call options are large than that for the put options, we can conclude that the investors maybe more concern about the put options and active more frequently for the put options

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<sup>7</sup>Since the observations for the bid orders of the OTM call options which have the  $K_4$  strike price at the jump day are very small and we can not exactly estimate it, there is no result for this case in Table 6 under the condition of not affecting our overall conclusions. Same case for the Table 8 and the Table 11.

when a jump happens than for the call options, which is opposite to the negative jump case. Third, by comparing all the ATM options and the ITM options, we find that all the branching ratios of the ATM option are higher than those of the ITM's, i.e.,  $\theta_A > \theta_I$ , which is consistent with the negative jump case. While there are no consistent conclusions when comparing the ATM options with the OTM options. Fourth, for the put options, we find that the branching ratios of the ask orders are less than the branching ratios of the bid orders, the ratios of the limit asks are large than the ratios of limit bids, and the ratios of MC asks are less than the ratios of MC bid for one day before and one day after the jump day, i.e.,  $\theta_A < \theta_B$ ,  $\theta_{Al} > \theta_{Bl}$ , and  $\theta_{Am} < \theta_{Bm}$ , while the contrary case for the jump day. There are no consistent results by comparing the branching ratios for the call options. Fifth, for the put options, by comparing the limit asks with the MC asks, we find that the branching ratios of the limit orders are large than those of the MC orders for one day before and one day after the jump day, i.e.,  $\theta_l^B > \theta_m^B$  and  $\theta_l^A > \theta_m^A$ , while are less than the branching ratio of MC orders for the jump day, i.e.,  $\theta_l^T < \theta_m^T$ . On the contrary, for the limit bids and the MC bids, the branching ratios of the limit orders are less than those of the MC orders for one day before and one day after the jump day, i.e.,  $\theta_l^B < \theta_m^B$  and  $\theta_l^A < \theta_m^A$ , while are large than the branching ratio of MC order for the jump day, i.e.,  $\theta_l^T > \theta_m^T$ . There are no consistent results for the call options too.

\* (Table 6 here)

## 4.2 Significance Test

To justify whether the structure of the investors's behavior do significantly change when an exogenous large jump happens, we propose a significance test by comparing the changes of the branching ratio one day before the jump with the jump day to check whether this changes are significant when there exists a sudden jump.

Recall the log-likelihood function for the univariate case of (4), we add a dummy variable and a new coefficient  $\Delta$  into the log-likelihood function to check whether the changes of the branching ratio when a jump happens are significant. The MLE function can be rewrite as

$$\ln \mathcal{L}(\{t_1, \dots, t_N\}) = -\mu t_N - \theta(1 + D\Delta) \sum_{i=1}^N (1 - e^{-\delta(t_N - t_i)}) + \sum_{i=1}^N \ln [\mu + \theta\delta(1 + D\Delta)A(i)],$$

where

$$A(i) = \begin{cases} \sum_{t_j < t_i} e^{-\delta(t_i - t_j)}, & \text{for } i \geq 2, \\ 0, & \text{otherwise,} \end{cases}$$

and  $D$  represents for the dummy variable which is 0 for the data of one day before the

jump and 1 for the data of the jump day.

We want to check whether  $\Delta$  is significantly different from 0. If  $D = 0$ , the branching ratio is  $\theta(1 + D\Delta) = \theta$  and if  $D = 1$ , the branching ratio is  $\theta(1 + D\Delta) = \theta(1 + \Delta)$  So we can use  $\Delta$  to represent the real change of the branching ratio when a sudden jump happens. If  $\Delta$  is significantly different from 0, we say there exists a significant structure change after the jump.  $\Delta > 0$  represents for the increment of the branching ratio, and  $\Delta < 0$  for the decrement of the branching ratio.

We use the following algorithm to calculate the mean and t-statistics for  $\Delta$ . We first set  $D = 0$  and estimate the branching ratio  $\theta_0$  for the data of one day before the jump. Then we set  $D$  to 1 to estimate the the branching ratio  $\theta_1$  and the changes of branching ratio  $\Delta_1$  for the data of the jump day. At last, we want to calculate the statistics of the real changes of branching ratio  $\Delta^*$ . Since the reference of branching ratio for the data of one day before the jump is  $\theta_0$  and the branching ratio for the data of the jump day is  $\theta_1(1 + \Delta_1)$ , while we need to get the branching ratio with the form of  $\theta_0(1 + \Delta^*)$ , we propose a conversion method to get  $\Delta^*$ :

$$\begin{aligned}
\theta_1(1 + \Delta_1) &= \theta_0 \times \frac{\theta_1}{\theta_0}(1 + \Delta_1) \\
&= \theta_0 \left( \frac{\theta_1}{\theta_0} + \frac{\theta_1}{\theta_0} \Delta_1 \right) \\
&= \theta_0 \left[ 1 + \left( \frac{\theta_1}{\theta_0} - 1 \right) + \frac{\theta_1}{\theta_0} \Delta_1 \right] \\
&= \theta_0(1 + \Delta^*).
\end{aligned} \tag{12}$$

Thus we can get that

$$\Delta^* = \left( \frac{\theta_1}{\theta_0} - 1 \right) + \frac{\theta_1}{\theta_0} \Delta_1,$$

the standard deviation is  $\sigma_{\Delta^*} = \frac{\theta_1}{\theta_0} \sigma_{\Delta_1}$ , and the  $t$ -statistic of  $\Delta^*$  is  $t_{\Delta^*} = \frac{\Delta^*}{\sigma_{\Delta^*}}$ .

Panel B of the Table 2 shows the results of significance test for the futures data. We find that, in both of the negative jump case and the positive jump case, the model have significant changes of the branching ratios when the jump happens, however, there are increments for the branching ratios in the negative jump case but are decrements in the positive jump case. The different results imply that the investors have different judgments when facing with the negative jump and positive jump.

\* (Table 7 here)

Table 7 shows the results of significance test for the options data in the negative jump case. We find that all of the  $\Delta^*$  for all the type of orders in both call options and put options are positive, and most of  $\Delta^*$  are significant at the 1% significance level, some are

significant at the 5% significance level, and a few are significant at the 10% significance level, imply that there exist significant increments of the branching ratios at the jump day after an overnight jump happens, which are consistent with the results of the future data.

\* (Table 8 here)

Table 8 shows the results of significance test for the option data in the positive jump case. Different from the negative jump case, we find that the number of  $\Delta^*$  that significant at the 1% significance level is less than the number that in the negative jump case, and with higher percent significant at the 5% and 10% significance level. The results that there are almost half positive  $\Delta^*$  and half negative  $\Delta^*$  imply that there exist both significant increments and significant decrements of the branching ratios at the jump day after an overnight jump happens, and thus we can not take conclusive results in the positive jump case. These inconclusive results maybe lead by the investors do not worry about the positive price jump and thus do not have a consistent responses when facing a positive jump.

**Summary of the Univariate Results** In summary, we find that the order arrivals in both of the futures and the option market have significant structure changes when there exist exogenous overnight price jumps, no matter in the negative jump case or in the positive jump case. There exist significant increments of the branching ratios at the jump day after a negative overnight jump, while significant decrements in positive jump case in the futures market. Things are more complicated in the options market, there are still have significant increment of the branching ratios when a jump happens for most of the types of orders: the ask orders, the limit asks, the MC asks, the bid orders, the limit bids, and the MC bids, for the types of options: the ATM calls, the ITM calls, the OTM calls, the ATM puts, the ITM puts, and the OTM puts with each kind of strike price:  $K_1$ ,  $K_2$ ,  $K_3$ , and  $K_4$  in the negative jump case, which are consistent with the results in futures market, but we can not draw a consistent conclusion for the increments or decrements of changes of branching ratio for all of the options in positive jump case. We think that maybe it because the investors do not worry much about the positive price jump since it can lead to excess return, **and the reasons to the asymmetric results for positive and negative cases here may be similar to the ones to the asymmetric results for the implied volatility from option prices.**

### 4.3 Bivariate Results of Index Futures and The Options

The above results we analyzed are based on the univariate Hawkes process. We find that many results in the options data are consistent with the tendency of results in the futures data. Since the options are the S&P 500 futures option that use S&P 500 Index future as

the underlying assets, we would wonder that whether the results and changes of branching ratio of the options data are led by the changes of futures other than the investors behavior to the option directly when facing an exogenous jump. Thus we use the bivariate Hawkes processes to fit both of the futures and options at the same time to check the mutual excitation, for which jumps of futures' process can elevate both intensities and hence induce jumps in the options' processes afterward.

We know that there exists six types of events: ask orders, limit asks, MC asks, bid orders, limit bids, MC bids, in the options. So we fit the futures data and each types of events by using the bivariate Hawkes processes, respectively. For the bivariate Hawkes process, we have four types of branching ratios:  $\theta_{11} = \frac{\beta_{11}}{\delta_1}$ ,  $\theta_{12} = \frac{\beta_{12}}{\delta_1}$ ,  $\theta_{21} = \frac{\beta_{21}}{\delta_2}$  and  $\theta_{22} = \frac{\beta_{22}}{\delta_2}$  where  $\theta_{11}$  and  $\theta_{22}$  represents for the self-excitation effect that the magnitude an order can lead to a next order of the same event,  $\theta_{12}$  represents for the mutual-excitation effect that the magnitude of the order of event 2 can lead to a next order of event 1, and  $\theta_{21}$  represents for the mutual-excitation effect that the magnitude of the order of event 1 can lead to a next order of event 2.

In this study, event 1 represents for the future orders, and event 2 represents for the option orders. Thus,  $\theta_{11}$  and  $\theta_{22}$  represents for the branching ratio of the self-excitation effect of the future orders and the option orders respectively,  $\theta_{12}$  represents for the branching ratio of the mutual-excitation effect that the option orders, such as the total ask orders, the limit-ask orders, the MC-ask orders, the total bid orders, the limit-bid orders, and the MC-bid orders, to the future orders, and  $\theta_{21}$  represents for the mutual-excitation effect that the future orders to the option orders.

\* (Table 9 here)

\* (Table 10 here)

Table 9 to Table 12 shows the estimated results of the four types of the branching ratios for the call options and the put options in the negative jump case and in the positive jump case, respectively. We find that almost all the  $\theta_{21}$  are rather small compare to the  $\theta_{22}$ , which imply that there are little mutual-excitation effects of the futures on the options compare to the self-excitation effects of the options itself. Also, almost all the  $\theta_{11}$  are larger than the  $\theta_{12}$ . Both of the self-excitation parameters are larger than the cross-excitation counterparts suggest that although submitted orders on each type of orders would induce a overall increase in order arrivals, they are most likely to induce more orders of the same type. Moreover, we find that  $\theta_{21} + \theta_{22}$  are almost equal to the branching ratios in the univariate Hawkes cases for each types of orders with each strike price for all the call and put options, which indicates that the total effects of the self-excitation and

the cross-excitation in the bivariate Hawkes processes are equal to self-excitation effect in the univariate Hawkes process for each types of orders in the option markets. These also justify our results in the univariate cases and confirm that the cross-effects are negligible.

\* (Table 11 here)

\* (Table 12 here)

## 5 Concluding Remarks

In our current study, we focus on the statistical modeling of the dynamics of limit-order books in the high-frequency S&P 500 Index future and future option market by employing the Hawkes processes. We complement the literature by providing evidences of how an exogenous large overnight jump would affect the order arrivals of the futures and options. A special feature of our study is that we use the branching ratio  $\theta$ , which is the ratio of the time scale  $\beta$  to the time decay  $\delta$ , to measure the magnitude of trading contagion, and analyze the branching ratio changes for all of the total orders, limit orders, market orders and cancelations, on both the bid and the ask sides in the option markets, in order to investigate whether there is a significant structure change after the jump.

The contributions from our study are manifold. Firstly, we find that the order arrivals in both of the future and the option markets have significant structure changes when there exist exogenous overnight price jumps. Secondly, there exist significant increments of the branching ratios at the jump day after a negative overnight jump, while significant decrements in the positive jump case in future markets. Thirdly, in the option markets, there are still have significant increments of the branching ratios when a jump happens for most of the types of orders across different strike prices in the negative jump case, while have no conclusive results for the positive jump case. Fourthly, most of the branching ratios of the call options are larger than those of the put options in the negative jump case while are less than those of the put options, especially for the ATM and ITM options in the positive jump case. Fifthly, all the branching ratios of the ATM option are higher than those of the ITM's in both the negative jump and positive jump cases. Sixthly, the branching ratios of the limit orders are large than those of the MC orders for most of the call and put options in the negative jump case, while things are more complicated in the positive jump case. This phenomenon is probably due to the change of the volatility. As asymmetric volatility refers to the volatility changes in the opposite direction of equity return, traders actually encounter larger hedging uncertainty when price drops. The panic drives traders to rely the information of previous trading more, the herding behavior naturally emerges. We also use the bivariate Hawkes processes to fit both of the

futures and options at the same time to check the mutual excitation and find that there are little mutual-excitation effects of the futures on the options compare to the self-excitation effects of the options itself.

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**Table 1:** The Statistics of the Overnight Price Jumps

RIC	Date	Time	Type	Price	Return(%)
SPM0	May 10, 2010	8:00:02	Trade	1155.3	4.1846
SPM0	May 25, 2010	8:00:01	Trade	1043.6	-2.5917
SPM0	Jun 4, 2010	8:00:07	Trade	1080.9	-2.0891
SPM1	Mar 15, 2011	8:00:16	Trade	1257.2	-2.9623
SPU1	Aug 8, 2011	8:00:27	Trade	1165.8	-2.6411
SPU1	Aug 18, 2011	8:00:10	Trade	1162	-2.3390
SPZ1	Sep 22, 2011	8:00:07	Trade	1123.5	-2.8257
SPZ1	Oct 27, 2011	8:00:27	Trade	1267.9	2.3986
SPZ1	Nov 9, 2011	8:00:02	Trade	1246.4	-2.1235
SPZ1	Nov 28, 2011	8:00:10	Trade	1187.7	5.5431
SPZ1	Nov 30, 2011	8:00:01	Trade	1232.5	3.0062

Note: This table shows the statistics of the overnight price jump in two years of 2010 and 2011 for the futures data. An exogenous overnight jump is detected if the absolute overnight return is large than 2%.

**Table 2:** The Results of the Index Futures

<b>Panel A: Branching Ratio</b>			
	B	T	A
Negative jump case	0.6417	0.8228	0.7805
Positive jump case	0.8531	0.7643	0.1926
<b>Panel B: Significance Test</b>			
	Negative jump case		Positive jump case
$\Delta$	0.2822***		-0.1042***
	(17.196)		(-6.6773)

Note: Panel A shows the branching ratio of the negative jump case and the positive jump case for the futures data. B represents for the day before the jump day, T represents for the jump day, and A represents for the day after the jump day. Panel B shows the results of the significance test from the day before the jump day to the jump day. (\*), (\*\*) and (\*\*\*) are significance levels at 10%, 5% and 1%, respectively. Values in the brackets are the t-statistic values.

**Table 3:** Trade Number of Three Days around the Jump: 2010.5.25

		Call Option									Put Option								
		Before			The Day			After			Before			The Day			After		
		ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM
$K_1$	A	690	589	479	693	614	421	659	507	387	671	367	391	705	465	430	636	313	566
	$A_L$	321	264	227	288	263	186	294	215	211	283	172	183	292	223	221	246	121	272
	$A_M$	369	325	252	405	351	235	365	292	176	388	195	208	413	242	209	390	192	294
	B	705	542	351	617	448	533	639	454	363	671	415	418	599	391	489	533	323	616
	$B_L$	301	227	143	225	158	242	278	182	180	284	198	198	200	153	202	212	133	313
	$B_M$	404	315	208	392	290	291	361	272	183	387	217	220	399	238	287	321	190	303
$K_2$	A	712	547	325	656	545	126	625	416	343	740	510	470	732	595	268	655	414	596
	$A_L$	337	225	180	274	234	67	272	185	181	313	216	221	321	260	133	258	163	283
	$A_M$	375	322	145	382	311	59	353	231	162	427	294	249	411	335	135	397	251	313
	B	663	598	239	591	403	274	563	373	407	733	567	402	672	430	405	643	360	625
	$B_L$	270	240	121	216	131	140	236	144	195	311	242	180	247	139	142	272	161	315
	$B_M$	393	358	118	375	272	134	327	229	212	422	325	222	425	291	263	371	199	310
$K_3$	A	670	510	336	619	407	110	535	258	332	759	523	508	727	614	334	661	456	314
	$A_L$	314	225	189	257	168	55	234	115	176	319	223	230	320	267	157	267	183	165
	$A_M$	356	285	147	362	239	55	301	143	156	440	300	278	407	347	177	394	273	149
	B	649	462	411	520	325	532	595	223	260	746	509	493	661	473	393	654	449	357
	$B_L$	256	196	204	179	119	274	247	84	144	319	229	219	245	143	149	272	180	189
	$B_M$	393	266	207	341	206	258	348	139	116	427	280	274	416	330	244	382	269	168
$K_4$	A	644	434	404	600	153	234	532	221	105	764	644	461	772	643	438	699	533	292
	$A_L$	286	185	208	247	66	120	221	108	58	325	271	199	349	287	209	289	224	122
	$A_M$	358	249	196	353	87	114	311	113	47	439	373	262	423	356	229	410	309	170
	B	637	418	496	467	280	131	545	208	39	750	623	415	673	592	335	670	438	249
	$B_L$	262	179	228	161	103	74	231	100	20	343	275	170	261	207	112	294	201	101
	$B_M$	375	239	268	306	177	57	314	108	19	407	348	245	412	385	223	376	237	148

Note: This table shows the trade number of three days around the negative jump day: May 25, 2010, for the options data. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders. Before represents for the day before the jump day, The Day represents for the jump day, and After represents for the day after the jump day.  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  represent for the four strike price that we extracted from each ATM, ITM and OTM types for call and put options, respectively.

**Table 4:** Branching Ratio of Three Days around the Jump: 2010.5.25 with the Univariate Hawkes Process

	Call Option									Put Option									
	ATM			ITM			OTM			ATM			ITM			OTM			
	B	T	A	B	T	A	B	T	A	B	T	A	B	T	A	B	T	A	
$K_1$	A	0.3233	0.5795	0.7494	0.1292	0.6114	0.2954	0.5511	0.8368	0.7698	0.6746	0.7567	0.8142	0.1150	0.3829	0.4056	0.8465	0.956	0.0710
	$A_L$	0.2418	0.5867	0.7140	0.1591	0.5285	0.2833	0.3403	0.7988	0.6220	0.5544	0.6712	0.7291	0.3618	0.5553	0.3533	0.6445	0.8093	0.0918
	$A_M$	0.2063	0.3267	0.5798	0.1012	0.4493	0.2755	0.4917	0.7085	0.5962	0.4953	0.5656	0.7175	0.7998	0.3908	0.3524	0.8183	0.7085	0.0207
	B	0.5869	0.6397	0.7761	0.2578	0.3870	0.1186	0.6549	0.5912	0.7451	0.6208	0.5419	0.7400	0.1110	0.3108	0.4955	0.6090	0.7409	0.2149
	$B_L$	0.4890	0.6302	0.6931	0.2335	0.4844	0.2599	0.4868	0.5623	0.5386	0.4921	0.4298	0.7281	0.1474	0.5273	0.4627	0.5041	0.6839	0.2868
	$B_M$	0.3693	0.4287	0.5843	0.3171	0.4005	0.1742	0.5876	0.2278	0.5248	0.4842	0.4309	0.5854	0.0347	0.0723	0.3838	0.4521	0.5101	0.5143
$K_2$	A	0.3592	0.8102	0.8514	0.1786	0.5421	0.5922	0.7064	0.7918	0.7515	0.4962	0.7042	0.7931	0.3151	0.3638	0.5198	0.6981	0.8087	0.3160
	$A_L$	0.3265	0.6793	0.7528	0.1511	0.4734	0.6083	0.5824	0.7026	0.5872	0.5666	0.6935	0.6420	0.3125	0.4094	0.4355	0.6090	0.8213	0.0117
	$A_M$	0.2425	0.7286	0.7031	0.2046	0.3449	0.3721	0.6644	0.5497	0.5813	0.3344	0.4120	0.6675	0.0514	0.1222	0.3627	0.5592	0.7035	0.3314
	B	0.2890	0.6326	0.7101	0.5039	0.7561	0.4541	0.5601	0.7361	0.7219	0.2496	0.6086	0.7607	0.5076	0.5610	0.5348	0.6322	0.6766	0.2161
	$B_L$	0.3490	0.5849	0.6388	0.1406	0.4984	0.3223	0.2591	0.6248	0.4843	0.1517	0.6201	0.7068	0.0838	0.4643	0.6460	0.5430	0.7009	0.0138
	$B_M$	0.1372	0.4697	0.5946	0.1255	0.3234	0.3955	0.5046	0.5837	0.5013	0.2131	0.4178	0.6298	0.0640	0.2654	0.2480	0.4850	0.5892	0.0461
$K_3$	A	0.4833	0.8421	0.6854	0.4523	0.5097	0.8416	0.8293	0.9113	0.5210	0.3108	0.7118	0.7276	0.3823	0.4639	0.3920	0.5093	0.8147	0.5918
	$A_L$	0.3407	0.7268	0.5720	0.3510	0.4675	0.7532	0.7180	0.8788	0.3120	0.3242	0.6801	0.5666	0.1968	0.4371	0.4521	0.4956	0.8440	0.4766
	$A_M$	0.4117	0.7304	0.5722	0.1577	0.3073	0.6454	0.6868	0.8559	0.1875	0.1628	0.4099	0.5195	0.0524	0.1414	0.1743	0.1995	0.7202	0.3246
	B	0.6603	0.7351	0.7758	0.1483	0.5872	0.4049	0.8281	0.5119	0.5662	0.7045	0.5691	0.5969	0.5414	0.6368	0.3658	0.5105	0.7531	0.6451
	$B_L$	0.5622	0.6985	0.6642	0.3178	0.5972	0.4587	0.6706	0.4739	0.4610	0.5212	0.5607	0.5518	0.1319	0.4423	0.3869	0.4826	0.6324	0.5336
	$B_M$	0.5311	0.5998	0.7442	0.1826	0.3305	0.2314	0.6499	0.7491	0.0205	0.6188	0.4425	0.4497	0.0364	0.0763	0.2489	0.2823	0.6848	0.4589
$K_4$	A	0.5292	0.7364	0.5515	0.3058	0.6333	0.6940	0.6824	0.8424	0.5737	0.2485	0.7286	0.4206	0.5394	0.7552	0.0811	0.4968	0.7699	0.8187
	$A_L$	0.4243	0.5902	0.4116	0.1971	0.5446	0.5435	0.6205	0.7213	0.3967	0.3995	0.7274	0.3749	0.2299	0.2860	0.0941	0.5248	0.7274	0.8152
	$A_M$	0.3428	0.5547	0.4080	0.2843	0.4898	0.5361	0.4770	0.7489	0.5385	0.1064	0.4470	0.4064	0.0873	0.1200	0.0439	0.2574	0.6708	0.7083
	B	0.5486	0.7109	0.7119	0.3015	0.5572	0.5568	0.7679	0.9490	0.5270	0.3122	0.3774	0.5332	0.1190	0.1580	0.4288	0.7063	0.7677	0.8307
	$B_L$	0.4602	0.6606	0.6355	0.2350	0.5272	0.5249	0.4887	0.8827	0.0603	0.2283	0.4873	0.3000	0.0518	0.4111	0.5669	0.5162	0.7881	0.6960
	$B_M$	0.3189	0.4967	0.4586	0.2177	0.3243	0.3132	0.5450	0.9071	0.5413	0.2956	0.1635	0.3890	0.4042	0.4979	0.1867	0.6457	0.8399	0.7680

Note: This table shows the estimated results of the branching ratios for the three days around the negative jump day: May 25, 2010, for the options data by using the univariate Hawkes process. From the row of the table, B represents for the day before the jump day, T represents for the jump day, and A represents for the day after the jump day. From the column of the table, A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  represent for the four strike price we extracted from each ATM, ITM and OTM types for call and put options, respectively.

**Table 5: Trade Number of Three Days around the Jump: 2011.10.27**

		Call Option									Put Option								
		Before			The Day			After			Before			The Day			After		
		ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM	ITM	ATM	OTM
$K_1$	A	322	364	277	421	332	208	286	204	184	354	152	210	349	221	291	199	228	239
	$A_L$	158	196	144	201	157	94	137	103	93	181	88	105	198	121	137	105	123	120
	$A_M$	164	168	133	220	175	114	149	101	91	173	64	105	151	100	154	94	105	119
	B	307	313	250	401	315	254	265	218	198	320	167	181	305	230	158	208	218	121
	$B_L$	148	139	127	213	173	151	123	112	107	154	97	99	134	111	82	102	107	64
	$B_M$	159	174	123	188	142	103	142	106	91	166	70	82	171	119	76	106	111	57
$K_2$	A	458	258	177	420	270	226	276	208	70	426	172	253	369	144	221	226	89	78
	$A_L$	228	134	92	193	127	88	126	113	34	231	111	136	202	83	118	113	49	41
	$A_M$	230	124	85	227	143	138	150	95	36	195	61	117	167	61	103	113	40	37
	B	453	248	116	410	269	202	274	173	80	368	186	249	338	125	133	250	96	109
	$B_L$	213	137	69	226	169	132	138	95	41	170	119	154	155	66	75	122	46	56
	$B_M$	240	111	47	184	100	70	136	78	39	198	67	95	183	59	58	128	50	53
$K_3$	A	424	232	118	423	215	39	265	182	111	390	235	253	191	241	177	282	118	82
	$A_L$	213	130	53	203	105	19	126	104	52	190	158	130	118	146	97	157	79	46
	$A_M$	211	102	65	220	110	20	139	78	59	200	77	123	73	95	80	125	39	36
	B	423	302	73	389	197	44	266	136	83	387	274	248	189	206	223	278	146	138
	$B_L$	204	201	43	217	128	24	135	78	53	192	193	129	76	109	124	126	87	67
	$B_M$	219	101	30	172	67	20	131	58	30	195	81	119	113	97	99	152	59	71
$K_4$	A	374	178	34	347	293	22	279	171	40	364	250	252	284	294	207	143	132	210
	$A_L$	196	117	19	158	145	9	126	93	24	187	143	120	164	169	119	78	81	118
	$A_M$	178	61	15	189	148	13	153	78	16	177	107	132	120	125	88	65	51	92
	B	357	183	21	356	312	6	278	84	83	365	264	220	294	290	197	145	152	223
	$B_L$	162	123	11	198	179	4	146	48	41	179	144	112	142	148	101	67	76	102
	$B_M$	195	60	10	158	173	2	132	36	42	186	120	108	152	142	96	78	76	121

Note: This table shows the trade number of three days around the positive jump day: October 27, 2011, for the options data. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders. Before represents for the day before the jump day, The Day represents for the jump day, and After represents for the day after the jump day.  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  represent for the four strike price we extracted from each ATM, ITM and OTM types for call and put options, respectively.

**Table 6:** Branching Ratio of Three Days around the Jump: 2011.10.27 with the Univariate Hawkes Process

	Call Option									Put Option									
	ATM			ITM			OTM			ATM			ITM			OTM			
	B	T	A	B	T	A	B	T	A	B	T	A	B	T	A	B	T	A	
$K_1$	A	0.5228	0.4564	0.4900	0.9925	0.3907	0.5605	0.5751	0.7231	0.5301	0.5756	0.5118	0.5064	0.5824	0.6488	0.5595	0.6578	0.5864	0.6861
	$A_L$	0.5603	0.6793	0.7231	0.9682	0.6127	0.6524	0.6634	0.8283	0.7807	0.8048	0.7073	0.7466	0.6265	0.5194	0.7927	0.7551	0.7323	0.7728
	$A_M$	0.5954	0.5669	0.6983	0.9534	0.6341	0.7209	0.7094	0.7849	0.7544	0.7930	0.7715	0.7255	0.6783	0.7727	0.7297	0.7259	0.6696	0.7285
	B	0.6721	0.6164	0.5369	0.9897	0.5118	0.5686	0.6179	0.5517	0.5194	0.5740	0.4242	0.4873	0.6006	0.5151	0.5409	0.5033	0.3819	0.5390
	$B_L$	0.7095	0.6118	0.7343	0.9528	0.5383	0.7640	0.7197	0.6607	0.7361	0.8580	0.7459	0.6964	0.7187	0.8034	0.6941	0.7265	0.7351	0.7242
	$B_M$	0.6149	0.7567	0.7259	0.9653	0.7037	0.6345	0.6848	0.7700	0.7466	0.7970	0.6974	0.7672	0.6580	0.4897	0.7740	0.7680	0.5301	0.8063
$K_2$	A	0.5027	0.4209	0.6159	0.5312	0.4892	0.3977	0.7001	0.9336	0.5436	0.7390	0.5919	0.6565	0.5428	0.6277	0.6128	0.6117	0.8595	0.5179
	$A_L$	0.6722	0.7540	0.7011	0.6090	0.6533	0.6999	0.7775	0.9077	0.6987	0.8347	0.7598	0.8074	0.6450	0.5736	0.7738	0.7106	0.7664	0.7166
	$A_M$	0.7462	0.5826	0.7647	0.4983	0.6349	0.6863	0.7891	0.9300	0.7803	0.7894	0.8861	0.7685	0.5853	0.7669	0.7515	0.7108	0.8237	0.7097
	B	0.4521	0.5809	0.6670	0.6703	0.5631	0.4965	0.5638	0.6289	0.5482	0.6089	0.7896	0.6950	0.6116	0.5442	0.5284	0.5767	0.4993	0.5525
	$B_L$	0.7098	0.6520	0.7804	0.6450	0.5994	0.6952	0.7407	0.7991	0.7227	0.7888	0.8718	0.7835	0.6580	0.7814	0.7349	0.6673	0.7680	0.7267
	$B_M$	0.7480	0.8317	0.7763	0.6449	0.7116	0.6690	0.7454	0.7704	0.7310	0.7998	0.8920	0.8449	0.6402	0.5895	0.7322	0.7612	0.6789	0.8328
$K_3$	A	0.7530	0.6419	0.6715	0.5045	0.4480	0.3939	0.5011	0.2826	0.6502	0.6413	0.5640	0.5765	0.6684	0.9809	0.4327	0.5665	0.8060	0.7437
	$A_L$	0.7662	0.6781	0.7586	0.5154	0.6124	0.7114	0.7278	0.6021	0.7804	0.7824	0.5847	0.7845	0.6843	0.9692	0.6963	0.7615	0.8497	0.9227
	$A_M$	0.7959	0.7196	0.8300	0.5954	0.6017	0.7234	0.7412	0.6737	0.7820	0.8526	0.7824	0.7027	0.6329	0.8622	0.7008	0.7203	0.8537	0.7817
	B	0.7400	0.5811	0.6069	0.5095	0.5803	0.3572	0.5838	0.9858	0.6143	0.6538	0.4712	0.7345	0.6725	0.9961	0.4048	0.6006	0.7884	0.6180
	$B_L$	0.7314	0.7881	0.7445	0.5834	0.5256	0.7116	0.7505	0.6062	0.7817	0.6345	0.7417	0.8307	0.6297	0.8554	0.6758	0.7355	0.7721	0.7906
	$B_M$	0.8400	0.9172	0.7874	0.6067	0.6575	0.6933	0.7019	0.6181	0.8037	0.7368	0.7408	0.8407	0.6651	0.9592	0.6833	0.7712	0.6946	0.7962
$K_4$	A	0.7383	0.4202	0.8085	0.5882	0.5402	0.3969	0.6572	0.5577	0.5362	0.6209	0.5197	0.6329	0.6169	0.4149	0.5044	0.7243	0.7906	0.5293
	$A_L$	0.8283	0.6542	0.8385	0.5899	0.6776	0.6760	0.6064	0.9770	0.6342	0.8108	0.6286	0.8287	0.6445	0.5403	0.9251	0.7812	0.8249	0.7806
	$A_M$	0.7951	0.6341	0.8208	0.5952	0.6482	0.6447	0.5979	0.7174	0.6096	0.6699	0.7913	0.7477	0.6077	0.7464	0.7638	0.7795	0.8607	0.7787
	B	0.7022	0.6546	0.6189	0.5050	0.6193	0.6001	0.1449		0.6962	0.7076	0.5051	0.7126	0.6151	0.3474	0.5118	0.7378	0.4520	0.6230
	$B_L$	0.8385	0.6319	0.7183	0.6352	0.5893	0.7136	0.5423	0.6453	0.7851	0.7704	0.7183	0.7884	0.5905	0.7398	0.7611	0.7993	0.6835	0.7569
	$B_M$	0.7811	0.6312	0.7446	0.6026	0.7474	0.7242	0.5473	0.8750	0.7921	0.7565	0.6508	0.8673	0.6453	0.5627	0.8835	0.8121	0.7413	0.7281

Note: This table shows the estimated results of the branching ratios of three days around the positive jump day: October 27, 2011, for the options data by using the univariate Hawkes process. From the row of the table, B represents for the day before the jump day, T represents for the jump day, and A represents for the day after the jump day. From the column of the table, A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  represent for the four strike price we extracted from each ATM, ITM and OTM types for call and put options, respectively.

**Table 7:** Significant Test of the Branching Ratio Changes from 5.24 to 5.25

		Call Option			Put Option		
		ATM	ITM	OTM	ATM	ITM	OTM
$K_1$	A	0.7925*** (13.479)	3.7322*** (26.555)	0.5184*** (11.733)	0.1217*** (3.6046)	2.3296*** (17.191)	0.1294*** (4.5273)
	$A_L$	1.4264*** (11.804)	2.3218*** (13.625)	1.3473*** (12.560)	0.2107*** (3.6054)	0.5348*** (7.0825)	0.2557*** (4.9187)
	$A_M$	0.5836*** (5.8312)	3.4266*** (16.027)	0.4409*** (6.7849)	0.1419** (2.3496)	-0.5114*** (-19.784)	-0.1342*** (-3.2421)
	B	0.0890** (2.3399)	0.5012*** (7.7640)	-0.0973*** (-3.1230)	-0.1271*** (-3.3860)	1.8000*** (12.857)	0.2166*** (5.9342)
	$B_L$	0.28888*** (3.6912)	1.0745*** (8.3948)	0.1550** (2.5146)	-0.1266* (-1.7917)	2.5773*** (11.6204)	0.3567*** (5.2163)
	$B_M$	0.1608** (2.3524)	0.2630*** (3.9217)	-0.6123*** (-18.430)	-0.1101* (-1.9207)	1.0836*** (3.9014)	0.1283** (2.1658)
$K_2$	A	1.2556*** (21.165)	2.0353*** (20.197)	0.1209* (1.9399)	0.4192*** (10.4003)	0.1546*** (3.2334)	0.1584*** (3.6666)
	$A_L$	1.0806*** (11.050)	2.1330*** (12.029)	1.4114*** (9.0187)	0.2240*** (4.1969)	0.3101*** (4.1020)	0.3486*** (4.8772)
	$A_M$	2.004*** (17.465)	0.6857*** (6.9417)	-0.1726* (-1.8900)	0.2321*** (3.3574)	1.3774*** (5.7250)	0.2585*** (3.4358)
	B	1.1889*** (14.369)	0.5000*** (12.358)	0.3412*** (5.9477)	1.4383*** (15.688)	0.1052*** (2.9653)	0.0702* (1.8498)
	$B_L$	0.6759*** (5.6885)	2.5448*** (11.635)	0.2064** (2.0127)	3.0877*** (11.410)	4.5406*** (13.545)	0.2908*** (3.8054)
	$B_M$	2.4235*** (12.397)	1.5769*** (9.886)	0.1567* (1.9360)	0.9606*** (8.2483)	3.1469*** (11.621)	0.2148*** (3.5800)
$K_3$	A	0.7423*** (14.542)	0.1269*** (3.1457)	0.0989* (1.8482)	1.2902*** (20.322)	0.2134*** (4.9690)	0.5996*** (11.190)
	$A_L$	1.2389*** (10.662)	0.3328*** (4.2247)	0.2240** (2.3132)	1.0978*** (11.949)	1.2210*** (9.9410)	0.7030*** (8.6179)
	$A_M$	0.7741*** (10.077)	0.9486*** (7.5710)	0.2462** (2.4361)	1.5178*** (10.901)	1.6985*** (6.7030)	2.6100*** (14.039)
	B	0.1133*** (2.7280)	2.9595*** (21.234)	-0.3818*** (-15.920)	-0.1921*** (-6.2940)	0.1762*** (5.0784)	0.4752*** (9.7617)
	$B_{LO}$	0.2424*** (2.9837)	0.8792*** (7.8498)	-0.2933*** (-7.1440)	0.0758 (1.0007)	2.3533*** (11.174)	0.3104*** (3.7880)
	$B_M$	0.1290** (2.0756)	0.8010*** (7.0362)	0.1526*** (3.1500)	-0.2849*** (-7.4054)	1.0961*** (4.1970)	1.4258*** (12.890)
$K_4$	A	0.3915*** (5.201)	1.0710*** (16.682)	0.2345*** (4.9078)	1.9320*** (24.679)	0.4000*** (11.9568)	0.5497*** (11.554)
	$A_L$	0.3910*** (2.7969)	1.7631*** (11.773)	0.1624** (2.2360)	0.8208*** (11.241)	0.2440*** (2.8387)	0.3861*** (5.9514)
	$A_M$	0.6181*** (4.2540)	0.7228*** (8.6329)	0.5700*** (5.7906)	3.2011*** (51.649)	0.3746*** (2.7250)	1.6061*** (12.920)
	B	0.2957*** (5.5253)	0.8481*** (11.889)	0.2358*** (4.1216)	0.2088*** (3.8564)	0.3277*** (3.5982)	0.0869** (2.2738)
	$B_L$	0.4355*** (4.1728)	1.2434*** (8.0677)	0.8062*** (6.7426)	1.1345*** (8.2531)	6.9363*** (13.699)	0.5267*** (5.6651)
	$B_M$	0.5575*** (5.2719)	0.4897*** (4.8411)	0.6644*** (5.3296)	-0.4469*** (-9.0780)	0.2331*** (4.1631)	0.3008*** (5.7091)

Note: This table shows the results of the significance test of the branching ratio changes of the negative jump day: May 25, 2010, for the options data. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  represent for the four strike price we extracted from each ATM, ITM and OTM types for call and put options, respectively.

**Table 8:** Significant Test of the Branching Ratio Changes from 10.26 to 10.27

		Call Option			Put Option		
		ATM	ITM	OTM	ATM	ITM	OTM
$K_1$	A	-0.1276*** (-2.7786)	-0.6063*** (29.652)	0.2730*** (4.5628)	-0.1109** (-2.1059)	0.1140*** (2.5844)	-0.1085*** (2.5906)
	$A_L$	0.2124*** (4.3469)	-0.3672*** (-15.032)	0.2486*** (4.5041)	-0.1211*** (-3.0977)	-0.1709*** (-4.6970)	-0.0302 (-0.7675)
	$A_M$	-0.0500 (-1.1504)	-0.3349*** (-14.026)	0.1064** (2.2957)	-0.0271 (-0.6125)	0.1392*** (3.3198)	-0.0775** (-2.0418)
	B	-0.0827** (-2.0486)	-0.4828*** (-21.171)	-0.1071** (-2.2812)	-0.2610*** (-5.3344)	-0.1424*** (-3.3065)	-0.2412*** (-3.7093)
	$B_L$	-0.1377*** (-3.8388)	-0.4350*** (18.644)	-0.0820** (-2.1812)	-0.1307*** (-3.3697)	0.1179*** (2.7817)	0.0118 (0.2234)
	$B_M$	0.2304*** (4.8394)	-0.2710*** (-10.413)	0.1244** (2.4699)	-0.1250*** (-3.1457)	-0.2558*** (-6.9854)	-0.3098*** (-6.5605)
	$K_2$	A	-0.1627*** (-3.1704)	-0.0791* (-1.9293)	0.3335*** (6.9281)	-0.1991*** (-3.7262)	0.1564*** (3.4155)
$A_L$		0.1217*** (2.6396)	0.0727*** (1.7914)	0.1675*** (3.4152)	-0.0897* (-1.9487)	-0.1107*** (-3.0735)	0.0785* (1.7294)
$A_M$		-0.2192*** (-5.9118)	0.2741*** (6.0946)	0.1786*** (4.6472)	0.1225** (2.0866)	0.3103*** (6.7081)	0.1588*** (3.2404)
B		0.2849*** (4.5158)	-0.1599*** (-4.6774)	0.1155* (1.9241)	0.2968*** (4.0291)	-0.1102*** (-2.6983)	-0.1342** (-1.9899)
$B_L$		-0.0814** (-2.2107)	-0.0707** (2.0568)	0.0788* (1.9189)	0.1052* (1.8890)	0.1875*** (4.3867)	0.1509** (2.4786)
$B_M$		0.1110** (2.3699)	0.1034*** (2.6183)	0.03354 (0.5954)	0.1150** (1.9618)	-0.0792** (-2.0643)	-0.1081* (-1.8093)
$K_3$		A	-0.1475*** (-3.4004)	-0.1120*** (-2.6776)	-0.4360*** (-3.7248)	-0.1205*** (-2.5811)	0.4675*** (8.4506)
	$A_L$	-0.1150*** (-2.6624)	0.1882*** (4.1248)	-0.1727 (-1.6036)	-0.2527*** (-7.2096)	0.4613*** (8.6203)	0.1158** (2.4482)
	$A_M$	-0.0958** (-2.3154)	0.0100 (0.2801)	-0.1414 (-1.4180)	-0.0823* (-1.9464)	0.3623*** (5.5176)	0.1852*** (3.3461)
	B	-0.2147*** (-4.8142)	0.1389*** (2.9832)	0.6886*** (4.8604)	-0.2793*** (-5.9345)	0.4812** (8.7096)	0.3127*** (5.5396)
	$B_L$	0.0700* (1.8435)	-0.0991*** (-2.6462)	-0.1923** (-2.0790)	0.1689*** (3.2118)	0.3584*** (5.5315)	0.0828* (1.9263)
	$B_M$	0.0900* (1.7864)	0.0837** (1.9627)	-0.1194 (-1.1306)	0.0054 (0.1138)	0.4422*** (8.7353)	-0.0993** (-2.2295)
	$K_4$	A	-0.4309*** (12.831)	-0.0816* (-1.9440)	-0.1514 (-0.9808)	-0.1603*** (-3.8333)	-0.3274*** (-8.0740)
$A_L$		-0.2102*** (-6.1603)	0.1487*** (3.2196)	0.6111*** (2.6962)	-0.2247*** (-7.0352)	-0.1617*** (-4.1122)	0.0559 (1.3492)
$A_M$		-0.2025*** (-5.7969)	0.0890** (2.1640)	0.1998 (1.1820)	0.1812*** (3.8742)	0.2282*** (4.3621)	0.1042** (2.1392)
B		-0.0806* (-1.748)	0.2263*** (4.5459)	()	-0.2862*** (-7.6656)	-0.4352*** (-11.642)	-0.3874*** (-9.2442)
$B_L$		-0.2464*** (-8.1942)	-0.0723* (-1.9473)	0.1899 (0.5369)	-0.0676* (-1.8288)	0.2528*** (5.1091)	-0.1449*** (-3.4021)
$B_M$		-0.1919*** (-5.1627)	0.2403*** (5.2220)	0.5988 (1.0020)	-0.1397*** (-3.7081)	-0.1280*** (-3.1034)	-0.0872** (-1.9774)

Note: This table shows the results of the significance test of the branching ratio changes of the positive jump day: October 27, 2011, for the options data. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  represent for the four strike price we extracted from each ATM, ITM and OTM types for call and put options, respectively.

**Table 9: Branching Ratio for Call Option of Three Days around the Negative Jump: 2010.5.25 of the Bivariate Hawkes Process**

		ATM				ITM				OTM									
		B		T		A		B		T		A							
		$\theta_{11}$ $\theta_{21}$	$\theta_{12}$ $\theta_{22}$																
$K_1$	A	0.6304	0.0578	0.8224	0.1242	0.8021	0.0193	0.8130	0.0517	0.8380	0.1686	0.7747	0.0501	0.6369	0.0132	0.7940	0.2979	0.7802	0.0000
	$A_L$	0.0416	0.2562	0.0000	0.5794	0.0046	0.8062	0.0560	0.6000	0.0000	0.6234	0.0340	0.2287	0.0107	0.5386	0.0000	0.8366	0.0000	0.7697
	$A_M$	0.6303	0.1153	0.8223	0.5206	0.7807	0.0000	0.6422	0.0319	0.8302	0.4470	0.7817	0.0001	0.6414	0.0207	0.8016	0.4207	0.7849	0.0002
	B	0.0037	0.2340	0.0000	0.5910	0.0000	0.7140	0.0000	0.1752	0.0000	0.5311	0.0023	0.2898	0.0075	0.3249	0.0000	0.7850	0.0042	0.6180
	$B_L$	0.6362	0.0444	0.8233	0.0002	0.7792	0.1019	0.7682	0.1978	0.8221	0.0000	0.7637	0.3260	0.6394	0.0157	0.8061	0.4043	0.7824	0.2271
	$B_M$	0.0241	0.1419	0.0000	0.3289	0.0122	0.5512	0.0216	0.1696	0.0000	0.4515	0.0215	0.2335	0.0114	0.4806	0.0000	0.7090	0.0000	0.5958
		0.6313	0.1090	0.8206	0.1339	0.7769	0.0105	0.6682	0.0392	0.8301	0.0210	0.7805	0.0000	0.6405	0.0308	0.7944	0.2295	0.9182	0.0645
		0.0000	0.5867	0.0000	0.6397	0.0031	0.7677	0.0120	0.1038	0.0003	0.3998	0.0498	0.0864	0.0000	0.6548	0.0007	0.5898	0.0047	0.7080
		0.6329	0.1456	0.8230	0.0000	0.7711	0.1862	0.6299	0.0989	0.8356	0.2099	0.7666	0.3148	0.6436	0.0382	0.8151	0.2050	0.7782	0.1413
		0.0000	0.4871	0.0000	0.6302	0.0067	0.6702	0.0145	0.1886	0.0056	0.4672	0.0307	0.2028	0.0001	0.4937	0.0000	0.4659	0.0014	0.5354
$K_2$	A	0.6414	0.0000	0.8189	0.0737	0.7801	0.0024	0.6413	0.0000	0.8224	0.0008	0.7761	0.0005	0.6374	0.1147	0.8324	0.2534	0.7799	0.0037
	$A_L$	0.0009	0.3581	0.0000	0.8106	0.0000	0.8512	0.0002	0.1772	0.0000	0.5449	0.0126	0.5751	0.0000	0.7552	0.0000	0.8052	0.0000	0.7477
	$A_M$	0.6392	0.1123	0.8171	0.3186	0.7810	0.0000	0.6357	0.0949	0.8077	0.4148	0.7828	0.0001	0.6430	0.0918	0.8231	0.3548	0.7764	0.0000
	B	0.0030	0.3190	0.0001	0.6745	0.0000	0.7519	0.0000	0.1455	0.0000	0.4741	0.0029	0.6088	0.0000	0.6266	0.0000	0.6996	0.0000	0.5867
	$B_L$	0.6414	0.0000	0.8228	0.0005	0.7696	0.2932	0.7589	0.0274	0.8236	0.0000	0.7776	0.0768	0.6340	0.1973	0.8259	0.1251	0.7777	0.0802
	$B_M$	0.0001	0.2429	0.0000	0.7291	0.0000	0.7036	0.0326	0.3349	0.0000	0.3443	0.0190	0.3308	0.0000	0.6645	0.0000	0.5502	0.0000	0.5691
		0.6437	0.0172	0.8249	0.0364	0.7661	0.2070	0.7543	0.1537	0.8208	0.0466	0.7796	0.0010	0.6412	0.0000	0.8228	0.0005	0.7923	0.1166
		0.0001	0.2964	0.0000	0.6323	0.0001	0.6952	0.0873	0.3490	0.0000	0.4007	0.0633	0.2797	0.0098	0.5605	0.0000	0.7362	0.0007	0.7195
		0.6403	0.0000	0.8090	0.0187	0.7669	0.4734	0.8297	0.2855	0.8165	0.0051	0.7764	0.2687	0.6414	0.0000	0.8234	0.0002	0.7736	0.1972
		0.0000	0.3484	0.0000	0.5842	0.0075	0.5885	0.0265	0.1165	0.0002	0.5019	0.0223	0.2023	0.0179	0.2786	0.0000	0.6242	0.0000	0.4842
$K_3$	A	0.6690	0.0283	0.8080	0.1297	0.7796	0.0001	0.6405	0.0005	0.8224	0.0093	0.8381	0.0246	0.6357	0.0592	0.8247	0.2184	0.7798	0.0004
	$A_L$	0.0056	0.4744	0.0000	0.8423	0.0000	0.6854	0.0000	0.1046	0.0000	0.5074	0.0006	0.8478	0.0003	0.8289	0.0000	0.9113	0.0000	0.5220
	$A_M$	0.6356	0.1449	0.8027	0.4228	0.7848	0.0002	0.7446	0.0425	0.8239	0.1936	0.9090	0.0601	0.6407	0.0682	0.8293	0.4976	0.7802	0.0001
	B	0.0091	0.3166	0.0000	0.7268	0.0000	0.5714	0.0124	0.2324	0.0000	0.4678	0.0001	0.7592	0.0015	0.7140	0.0000	0.8787	0.0000	0.3081
	$B_L$	0.6414	0.0000	0.8207	0.0264	0.7806	0.0000	0.7216	0.0657	0.8254	0.0012	0.7779	0.1714	0.6338	0.0911	0.8221	0.2602	0.8174	0.2770
	$B_M$	0.0000	0.4117	0.0000	0.7293	0.0000	0.5721	0.0006	0.1654	0.0034	0.3019	0.0000	0.6458	0.0000	0.6869	0.0000	0.8555	0.0293	0.6107
		0.6429	0.0016	0.8176	0.0429	0.7814	0.1268	0.7586	0.1415	0.8227	0.0356	0.7804	0.0000	0.6412	0.0311	0.8395	0.0000	0.7601	0.2810
		0.0000	0.6584	0.0000	0.7367	0.0000	0.7758	0.0886	0.3413	0.0000	0.5875	0.0350	0.3028	0.0000	0.8260	0.0000	0.5110	0.0000	0.5613
		0.6404	0.0019	0.8229	0.0003	0.7857	0.6580	0.6374	0.0406	0.8375	0.0570	0.7831	0.2826	0.6435	0.0566	0.8445	0.1803	0.7812	0.5677
		0.0000	0.5521	0.0000	0.6465	0.0000	0.6541	0.0035	0.3078	0.0000	0.5940	0.0119	0.4054	0.0000	0.6714	0.0384	0.4339	0.0000	0.4580
$K_4$	A	0.6604	0.0461	0.8189	0.0143	0.7661	0.0252	0.6424	0.0005	0.8109	0.1567	0.7689	0.0766	0.6411	0.0001	0.8194	0.0420	0.7690	0.1561
	$A_L$	0.0078	0.5177	0.0000	0.7368	0.0038	0.5157	0.0000	0.3029	0.0000	0.6334	0.0170	0.6603	0.0010	0.6827	0.0000	0.8412	0.0082	0.5238
	$A_M$	0.6349	0.0974	0.8232	0.0001	0.7804	0.0000	0.6390	0.1074	0.8203	0.3981	0.7800	0.0270	0.6403	0.0001	0.8147	0.0396	0.7780	0.0682
	B	0.0065	0.4187	0.0000	0.5907	0.0000	0.4117	0.0000	0.1972	0.0000	0.5434	0.0029	0.5017	0.0000	0.6207	0.0000	0.7235	0.0107	0.1673
	$B_L$	0.6416	0.0000	0.8392	0.0704	0.7809	0.0924	0.7435	0.0759	0.8228	0.0000	0.7793	0.1999	0.6396	0.0101	0.8231	0.0006	0.7675	1.1964
	$B_M$	0.0000	0.3430	0.0001	0.5235	0.0068	0.3946	0.0004	0.3513	0.0000	0.4897	0.0290	0.4626	0.0000	0.4766	0.0000	0.7489	0.0000	0.5407
		0.6297	0.0834	0.8145	0.0473	0.7806	0.0001	0.7215	0.0486	0.8229	0.0000	0.7680	0.1310	0.6366	0.0056	0.8294	0.0375	0.7804	0.0000
		0.0000	0.5486	0.0000	0.7018	0.0000	0.7118	0.0236	0.2915	0.0000	0.5570	0.0202	0.5110	0.0000	0.7695	0.0000	0.9498	0.0000	0.5251
		0.6361	0.1295	0.8212	0.0723	0.7738	0.2545	0.6412	0.0108	0.8411	0.0194	0.7603	0.4222	0.6374	0.0165	0.8227	0.0000	0.7642	0.3006
		0.0000	0.4603	0.0000	0.6606	0.0000	0.6356	0.0188	0.1328	0.0001	0.4464	0.0000	0.5247	0.0000	0.4836	0.0000	0.8823	0.0007	0.0639
	0.6365	0.0529	0.8380	0.2263	0.7799	0.0176	0.7652	0.1763	0.8223	0.0498	0.8207	0.2046	0.6440	0.0172	0.8170	0.2986	0.7807	0.0281	
	0.0000	0.3189	0.0027	0.5042	0.0000	0.4610	0.0440	0.1617	0.0000	0.3253	0.0319	0.3915	0.0000	0.5501	0.0000	0.9077	0.0000	0.5422	

Note: This table shows the estimated results of branching ratios of three days around the negative jump day: May 25, 2010, for the call options data by using bivariate Hawkes processes.  $\theta_{11}$  and  $\theta_{22}$  represents for the branching ratio of the self-excitation effect,  $\theta_{12}$  represents for the branching ratio of the mutual-excitation effect that event 2 to event 1, and  $\theta_{21}$  represents for the mutual-excitation effect that event 1 to event 2. In this study, event 1 represents for the future orders, event 2 represents for the option orders. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.

**Table 10: Branching Ratio for Put Option of Three Days around the Negative Jump: 2010.5.25 of the Bivariate Hawkes Process**

		ATM						ITM						OTM					
		B		T		A		B		T		A		B		T		A	
		$\theta_{11}$ $\theta_{21}$	$\theta_{12}$ $\theta_{22}$																
K <sub>1</sub>	A	0.6190	0.2932	0.8229	0.0000	0.7750	0.0011	0.6704	0.0778	0.8157	0.1463	0.7836	0.0010	0.6522	0.0422	0.8523	0.0570	0.7842	0.1515
		0.0083	0.6667	0.0000	0.7567	0.0000	0.8141	0.0786	0.6425	0.0000	0.3819	0.0000	0.3941	0.0014	0.8284	0.0000	0.9562	0.0421	0.6442
	A <sub>L</sub>	0.6816	0.0996	0.8115	0.1193	0.7673	0.2444	0.6401	0.0265	0.8202	0.0648	0.7861	0.3354	0.6413	0.0000	0.9042	0.0630	0.8397	0.1807
		0.0094	0.5472	0.0000	0.6713	0.0000	0.7291	0.0000	0.3558	0.0000	0.5548	0.0026	0.3263	0.0000	0.6445	0.0027	0.8030	0.0341	0.5813
	A <sub>M</sub>	0.7537	0.4875	0.8229	0.0000	0.7728	0.0000	0.7471	0.1337	0.8017	0.3061	0.7803	0.0000	0.6414	0.0000	0.8142	0.1875	0.8335	0.1604
		0.0135	0.3257	0.0000	0.5656	0.0012	0.7133	0.0027	0.1065	0.0030	0.3526	0.0000	0.3524	0.0016	0.8160	0.0000	0.7085	0.0202	0.4745
	B	0.6379	0.0309	0.8199	0.0835	0.7746	0.0519	0.6572	0.0127	0.8152	0.1354	0.7801	0.0000	0.6410	0.0002	0.8230	0.0000	0.8262	0.0277
		0.0000	0.6207	0.0000	0.5418	0.0000	0.7405	0.0007	0.0721	0.0001	0.3110	0.0080	0.4894	0.0000	0.6081	0.0000	0.7408	0.0209	0.6525
B <sub>L</sub>	0.6424	0.0690	0.8147	0.3269	0.7801	0.0001	0.6453	0.0684	0.8192	0.2799	0.7806	0.0000	0.6408	0.1272	0.8229	0.0000	0.8277	0.3000	
	0.0000	0.4936	0.0001	0.4579	0.0001	0.7271	0.0000	0.1469	0.0419	0.4462	0.0026	0.4612	0.0016	0.5029	0.0000	0.6838	0.1076	0.0717	
B <sub>M</sub>	0.6404	0.0096	0.8349	0.2167	0.7688	0.2342	0.7661	0.1623	0.7764	0.2315	0.7765	0.0935	0.6414	0.0000	0.8228	0.0000	0.8382	0.1804	
	0.0017	0.4794	0.0056	0.4990	0.0050	0.5735	0.0399	0.1916	0.0183	0.2007	0.0123	0.3685	0.0000	0.4530	0.0000	0.5107	0.0389	0.5965	
K <sub>2</sub>	A	0.6368	0.0064	0.7933	0.2071	0.7807	0.0000	0.7612	0.0156	0.8242	0.0003	0.8033	0.0244	0.6809	0.0025	0.8677	0.0421	0.7804	0.0000
		0.0100	0.4774	0.0000	0.7044	0.0000	0.7903	0.0410	0.7492	0.0000	0.3039	0.0000	0.2988	0.0000	0.7085	0.0008	0.8216	0.0000	0.3220
	A <sub>L</sub>	0.6412	0.0120	0.8078	0.1816	0.7469	0.4925	0.6406	0.0010	0.8248	0.0000	0.7783	0.2085	0.6413	0.0000	0.8228	0.0000	0.8392	0.1794
		0.0002	0.5600	0.0000	0.6947	0.0008	0.6378	0.0000	0.3136	0.0000	0.4093	0.0000	0.4371	0.0000	0.6090	0.0000	0.8219	0.0332	0.5911
	A <sub>M</sub>	0.6365	0.0740	0.8075	0.3081	0.7796	0.0001	0.7589	0.1216	0.7649	0.0004	0.7799	0.0001	0.6414	0.0000	0.8235	0.0000	0.9038	0.0624
		0.0085	0.3191	0.0000	0.4101	0.0018	0.6649	0.0191	0.1969	0.0233	0.1602	0.0000	0.3630	0.0000	0.5594	0.0000	0.7040	0.0514	0.4846
	B	0.6513	0.0006	0.9091	0.0965	0.7807	0.0993	0.7493	0.0061	0.8234	0.0002	0.7832	0.0000	0.6856	0.00017	0.8228	0.0000	0.7801	0.0000
		0.0001	0.1895	0.0332	0.5879	0.0000	0.7605	0.0531	0.7503	0.0000	0.3338	0.0000	0.5282	0.0001	0.6374	0.0000	0.6736	0.0000	0.2203
B <sub>L</sub>	0.6402	0.0409	0.8161	0.3588	0.7681	0.0309	0.7807	0.2269	0.8336	0.2531	0.7759	0.0000	0.6413	0.0088	0.8234	0.0000	0.8331	0.1509	
	0.0000	0.1516	0.0000	0.6247	0.0014	0.7085	0.0172	0.1339	0.0000	0.4652	0.0000	0.6365	0.0000	0.5436	0.0000	0.7020	0.0131	0.5858	
B <sub>M</sub>	0.6414	0.0000	0.8223	0.0000	0.7917	0.4471	0.7507	0.0670	0.8229	0.0000	0.7776	0.1630	0.6181	0.0132	0.8270	0.0608	0.8113	0.3084	
	0.0000	0.2128	0.0136	0.3896	0.0000	0.6311	0.0427	0.4613	0.0000	0.2652	0.0138	0.2096	0.0000	0.4620	0.0000	0.4614	0.1047	0.0685	
K <sub>3</sub>	A	0.6414	0.0000	0.8199	0.0051	0.7894	0.0213	0.6656	0.1168	0.7930	0.2321	0.7805	0.0000	0.6520	0.0208	0.8231	0.0000	0.7817	0.0001
		0.0000	0.3108	0.0022	0.7149	0.0000	0.7366	0.0146	0.4966	0.0298	0.4445	0.0066	0.3797	0.0277	0.4997	0.0000	0.8148	0.0000	0.5918
	A <sub>L</sub>	0.6414	0.0000	0.8233	0.0000	0.7883	0.3215	0.6286	0.1476	0.8297	0.0044	0.7785	0.3506	0.6393	0.0881	0.8211	0.0676	0.7808	0.0164
		0.0000	0.3242	0.0000	0.6799	0.0000	0.5667	0.0000	0.2003	0.0245	0.3834	0.0075	0.4020	0.0000	0.4956	0.0000	0.8446	0.0000	0.4768
	A <sub>M</sub>	0.6406	0.0150	0.8196	0.0160	0.7809	0.0000	0.7606	0.0902	0.7791	0.2558	0.7807	0.0000	0.6424	0.0064	0.8210	0.0074	0.8220	0.0229
		0.0000	0.1633	0.0124	0.3852	0.0001	0.5206	0.0430	0.4238	0.0004	0.1398	0.0000	0.1733	0.0185	0.1729	0.0000	0.7203	0.0011	0.2843
	B	0.6374	0.0000	0.8218	0.0211	0.7745	0.0268	0.7674	0.0020	0.8198	0.0487	0.7947	0.0180	0.6363	0.0595	0.8240	0.0013	0.7758	0.0774
		0.0001	0.7034	0.0000	0.5675	0.0057	0.5835	0.0638	0.7142	0.0000	0.2401	0.0065	0.3843	0.0000	0.5106	0.0000	0.7469	0.0000	0.1646
B <sub>L</sub>	0.6431	0.1015	0.7927	0.6391	0.7959	0.0104	0.6461	0.0966	0.8241	0.3553	0.7806	0.0000	0.6419	0.1812	0.8327	0.3324	0.7804	0.0000	
	0.0000	0.5219	0.0003	0.5482	0.0062	0.5188	0.0000	0.1307	0.0000	0.4420	0.0000	0.3824	0.0000	0.4819	0.0000	0.6324	0.0000	0.5338	
B <sub>M</sub>	0.6369	0.0000	0.8234	0.0000	0.7667	0.1470	0.7419	0.0466	0.8545	0.3422	0.7720	0.2126	0.6419	0.0002	0.8231	0.0034	0.7710	0.3610	
	0.0010	0.6151	0.0003	0.4420	0.0027	0.4323	0.0119	0.1700	0.0542	0.2288	0.0218	0.2098	0.0000	0.2821	0.0000	0.6851	0.0000	0.4594	
K <sub>4</sub>	A	0.7320	0.0832	0.8348	0.0000	0.7776	0.0006	0.7520	0.1812	0.7704	0.4179	0.7552	0.2577	0.6386	0.0212	0.8039	0.1467	0.7538	0.0836
		0.0262	0.4689	0.0001	0.7450	0.0000	0.4243	0.0411	0.7303	0.0291	0.7283	0.0336	0.9418	0.0000	0.4968	0.0055	0.7610	0.0000	0.8168
	A <sub>L</sub>	0.6432	0.0015	0.8220	0.0022	0.7816	0.2017	0.6415	0.0000	0.8203	0.0368	0.7722	0.2977	0.6061	0.0274	0.8057	0.2003	0.7495	0.7108
		0.0020	0.3901	0.0000	0.7280	0.0000	0.3426	0.0000	0.2256	0.0248	0.2460	0.0253	0.0709	0.0049	0.4913	0.0030	0.7165	0.0000	0.8142
	A <sub>M</sub>	0.7745	0.1378	0.8225	0.0004	0.7801	0.0000	0.7520	0.2207	0.8745	0.6058	0.8269	0.1560	0.6411	0.0136	0.8038	0.1815	0.7805	0.0000
		0.0166	0.0780	0.0000	0.4460	0.0000	0.4068	0.0101	0.1348	0.0027	0.1106	0.0000	0.0473	0.0002	0.2567	0.0098	0.6409	0.0056	0.6878
	B	0.6409	0.0095	0.8203	0.0281	0.8359	0.0050	0.7383	0.1499	0.8282	0.0554	0.7796	0.0000	0.6315	0.0694	0.8230	0.0000	0.7734	0.1373
		0.0277	0.2825	0.0000	0.3335	0.0001	0.5828	0.0349	0.7361	0.0029	0.1259	0.0125	0.4100	0.0000	0.7066	0.0000	0.7646	0.0000	0.8322
B <sub>L</sub>	0.6391	0.1157	0.7999	0.6604	0.7812	0.0000	0.7585	0.2311	0.8145	0.2336	0.8877	0.1866	0.6583	0.0267	0.7996	0.0263	0.7806	0.0000	
	0.0107	0.2013	0.0000	0.4871	0.0000	0.3021	0.0352	0.1015	0.0000	0.4148	0.0005	0.2294	0.0014	0.5147	0.0009	0.7434	0.0000	0.6960	
B <sub>M</sub>	0.6217	0.0028	0.8229	0.0000	0.7801	0.1342	0.8122	0.2219	0.8990	0.0998	0.7770	0.0755	0.6283	0.1259	0.8230	0.0000	0.7802	0.4240	
	0.0132	0.1858	0.0162	0.1221	0.0012	0.3866	0.0078	0.1150	0.0094	0.4868	0.0297	0.1032	0.0000	0.6478	0.0000	0.6229	0.0000	0.7680	

Note: This table shows the estimated results of branching ratios of three days around the negative jump day: May 25, 2010, for the put options data by using bivariate Hawkes processes.  $\theta_{11}$  and  $\theta_{22}$  represents for the branching ratio of the self-excitation effect,  $\theta_{12}$  represents for the branching ratio of the mutual-excitation effect that event 2 to event 1, and  $\theta_{21}$  represents for the mutual-excitation effect that event 1 to event 2. In this study, event 1 represents for the future orders, event 2 represents for the option orders. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.

**Table 11: Branching Ratio for Call option of Three Days around the Positive Jump: 2011.10.27 of the Bivariate Hawkes Process**

		ATM				ITM				OTM									
		B		T		A		B		T		A							
		$\theta_{11}$ $\theta_{21}$	$\theta_{12}$ $\theta_{22}$																
$K_1$	A	0.8495	0.0060	0.7569	0.0499	0.1915	0.0021	0.8571	0.0129	0.7618	0.0845	0.1919	0.0000	0.8476	0.0538	0.7645	0.0000	0.1921	0.0008
	$A_L$	0.0434	0.4627	0.0000	0.4587	0.0310	0.4690	0.0000	0.9957	0.0000	0.3897	0.0206	0.6519	0.0009	0.5726	0.0000	0.7231	0.0102	0.5370
	$A_M$	0.8530	0.0000	0.7642	0.0000	0.1919	0.0000	0.8531	0.0000	0.7644	0.0000	0.1917	0.0000	0.8531	0.0000	0.7439	0.0000	0.1918	0.0000
	B	0.0179	0.5597	0.0000	0.6797	0.0000	0.7231	0.0000	0.9682	0.0000	0.6139	0.0000	0.6519	0.0000	0.6633	0.0000	0.8261	0.0000	0.7807
	$B_L$	0.8530	0.0000	0.7644	0.0000	0.1919	0.0000	0.8531	0.0000	0.7618	0.0000	0.1920	0.0000	0.8531	0.0000	0.7635	0.0000	0.1960	0.0001
	$B_M$	0.0000	0.5954	0.0000	0.5669	0.0000	0.6982	0.0000	0.9534	0.0000	0.6458	0.0000	0.7218	0.0000	0.7318	0.0000	0.7838	0.0009	0.7586
		0.8528	0.0000	0.6696	0.1493	0.1886	0.0256	0.8739	0.0064	0.7574	0.1076	0.1920	0.0000	0.8531	0.0000	0.9023	0.3365	0.1917	0.0000
		0.0576	0.4728	0.0095	0.6063	0.0373	0.5016	0.0000	0.9795	0.0000	0.5119	0.0228	0.5447	0.0146	0.5929	0.0001	0.5463	0.0001	0.5203
		0.8529	0.0000	0.7642	0.0000	0.1907	0.0000	0.8418	0.0000	0.7644	0.0000	0.1918	0.0000	0.8543	0.0000	0.7641	0.0000	0.1836	0.0007
		0.0000	0.7095	0.0000	0.6115	0.0000	0.7349	0.0000	0.9469	0.0053	0.5427	0.0000	0.7642	0.0000	0.7197	0.0000	0.6614	0.0002	0.7451
$K_2$	A	0.8077	0.2150	0.7529	0.0797	0.1935	0.0567	0.8527	0.0133	0.7577	0.0656	0.1916	0.0000	0.8516	0.0357	0.7654	0.0206	0.1892	0.0440
	$A_L$	0.0262	0.4596	0.0000	0.4212	0.0198	0.6018	0.0432	0.4480	0.0000	0.4911	0.0245	0.3811	0.0087	0.7018	0.0000	0.9341	0.0000	0.5436
	$A_M$	0.8535	0.0000	0.7620	0.0000	0.1949	0.0000	0.8380	0.0000	0.7643	0.0000	0.1919	0.0000	0.8526	0.0000	0.7945	0.0000	0.1918	0.0000
	B	0.0000	0.7462	0.0000	0.5782	0.0000	0.7667	0.0046	0.6185	0.0128	0.6340	0.0000	0.6863	0.0000	0.7895	0.0000	0.9155	0.0000	0.7803
	$B_L$	0.8391	0.1146	0.7577	0.0763	0.1962	0.0256	0.8548	0.0020	0.6831	0.0867	0.1801	0.0000	0.8522	0.0001	0.7640	0.0000	0.1926	0.0254
	$B_M$	0.0281	0.4114	0.0000	0.5834	0.0143	0.6576	0.0303	0.5151	0.0047	0.5231	0.0243	0.4845	0.0133	0.5638	0.0000	0.6111	0.0049	0.5437
		0.8531	0.0000	0.7643	0.0000	0.1920	0.0000	0.8528	0.0000	0.7423	0.0000	0.1918	0.0000	0.8575	0.0001	0.7934	0.0000	0.1918	0.0000
		0.0000	0.7097	0.0000	0.6511	0.0000	0.7806	0.0000	0.5712	0.0110	0.6049	0.0000	0.6955	0.0003	0.7370	0.0000	0.8144	0.0000	0.7229
		0.8158	0.0001	0.7642	0.0000	0.1919	0.0000	0.8530	0.0000	0.7643	0.0000	0.1921	0.0001	0.8531	0.0000	0.7648	0.0000	0.1943	0.0009
		0.0052	0.7499	0.0000	0.8316	0.0000	0.7765	0.0258	0.5768	0.0000	0.7117	0.0000	0.6700	0.0038	0.7404	0.0000	0.7707	0.0044	0.7336
$K_3$	A	0.8532	0.0000	0.7650	0.0000	0.1934	0.0092	0.8346	0.1382	0.7306	0.1601	0.1925	0.0002	0.8383	0.2133	0.7592	0.4567	0.1842	0.0635
	$A_L$	0.0263	0.6810	0.0000	0.6420	0.0106	0.6683	0.0272	0.4682	0.0035	0.4680	0.0091	0.3862	0.0131	0.4738	0.0000	0.2845	0.0000	0.6501
	$A_M$	0.8536	0.0000	0.7641	0.0000	0.1919	0.0000	0.8528	0.0000	0.7643	0.0000	0.1920	0.0000	0.8535	0.0000	0.7592	0.0002	0.1919	0.0000
	B	0.0078	0.7823	0.0000	0.7757	0.0000	0.7585	0.0199	0.5477	0.0003	0.6127	0.0000	0.7114	0.0050	0.7236	0.0000	0.6021	0.0000	0.7804
	$B_L$	0.8499	0.0001	0.7640	0.0000	0.1919	0.0000	0.8532	0.0000	0.7644	0.0000	0.1918	0.0000	0.8560	0.0000	0.7838	0.2661	0.1920	0.0000
	$B_M$	0.0000	0.7989	0.0000	0.7195	0.0000	0.8300	0.0000	0.5959	0.0126	0.6011	0.0000	0.7238	0.0006	0.7424	0.0000	0.6759	0.0000	0.7820
		0.8512	0.0250	0.7591	0.0553	0.1918	0.0000	0.8427	0.0874	0.7598	0.0451	0.1913	0.0000	0.8323	0.4753	0.7643	0.0001	0.1899	0.0807
		0.0488	0.5826	0.0000	0.7950	0.0086	0.6071	0.0187	0.4905	0.0006	0.5125	0.0106	0.3534	0.0071	0.5627	0.0000	0.9848	0.0000	0.6143
		0.8533	0.0000	0.7525	0.0000	0.1919	0.0000	0.8527	0.0000	0.6726	0.0029	0.1933	0.0000	0.8542	0.0000	0.7640	0.0001	0.1921	0.0016
		0.0000	0.7318	0.0000	0.7791	0.0000	0.7445	0.0000	0.5804	0.0082	0.4684	0.0000	0.7122	0.0015	0.7524	0.0498	0.6043	0.0000	0.7818
$K_4$	A	0.8532	0.0000	0.7652	0.0000	0.1919	0.0000	0.8543	0.0000	0.7642	0.0000	0.1915	0.0000	0.8532	0.0001	0.7640	0.0002	0.1910	0.0001
	$A_L$	0.0102	0.8413	0.0000	0.9174	0.0000	0.7875	0.0247	0.5854	0.0000	0.6577	0.0000	0.6938	0.0022	0.6928	0.0000	0.6465	0.0001	0.8003
	$A_M$	0.8530	0.0000	0.7642	0.0000	0.1919	0.0000	0.8529	0.0002	0.7644	0.0000	0.1919	0.0000	0.8358	1.3388	0.7455	0.2910	0.1919	0.0000
	B	0.0084	0.7402	0.0000	0.4196	0.0000	0.8084	0.0495	0.5263	0.0000	0.5402	0.0266	0.3540	0.0000	0.6562	0.0009	0.5678	0.0000	0.5362
	$B_L$	0.8414	0.0001	0.7619	0.0000	0.1900	0.0001	0.8489	0.0000	0.7636	0.0000	0.1914	0.0000	0.8543	0.3465	0.7852	0.8702	0.1920	0.0001
	$B_M$	0.0016	0.8313	0.0000	0.6445	0.0000	0.8363	0.0148	0.5791	0.0000	0.6782	0.0000	0.6765	0.0029	0.6168	0.0000	0.9911	0.0000	0.6349
		0.8531	0.0000	0.7642	0.0000	0.1920	0.0000	0.8531	0.0000	0.7613	0.0000	0.1923	0.0000	0.8542	0.0749	0.7806	0.2558	0.1923	0.0000
		0.0000	0.7951	0.0000	0.6345	0.0000	0.8212	0.0000	0.5957	0.0038	0.6464	0.0000	0.6453	0.0025	0.6049	0.0014	0.7162	0.0000	0.6105
		0.8531	0.0000	0.7643	0.0000	0.1914	0.0315	0.8531	0.0000	0.7680	0.0330	0.1920	0.0000	0.8485	0.8759	0.0000	0.0000	0.1932	0.0006
		0.0055	0.7017	0.0000	0.6557	0.0005	0.6191	0.0477	0.5135	0.0234	0.6120	0.0098	0.5956	0.0002	0.1343	0.0000	0.0000	0.0001	0.6982
	0.8524	0.0000	0.7635	0.0000	0.1901	0.0014	0.8580	0.0000	0.7652	0.0000	0.1917	0.0000	0.8572	0.7020	0.7833	2.7792	0.1928	0.0000	
	0.0000	0.8392	0.0000	0.6319	0.0000	0.7189	0.0000	0.6340	0.0011	0.6012	0.0000	0.7136	0.0001	0.5342	0.0002	0.6563	0.0000	0.7815	
	0.8634	0.0000	0.7631	0.0000	0.1918	0.0000	0.8531	0.0000	0.7586	0.0000	0.2015	0.0012	0.8608	1.2304	0.7926	5.7511	0.1917	0.0000	
	0.0000	0.7823	0.0000	0.7849	0.0000	0.7444	0.0246	0.5851	0.0000	0.7464	0.0001	0.7238	0.0000	0.5468	0.0000	0.8792	0.0000	0.7918	

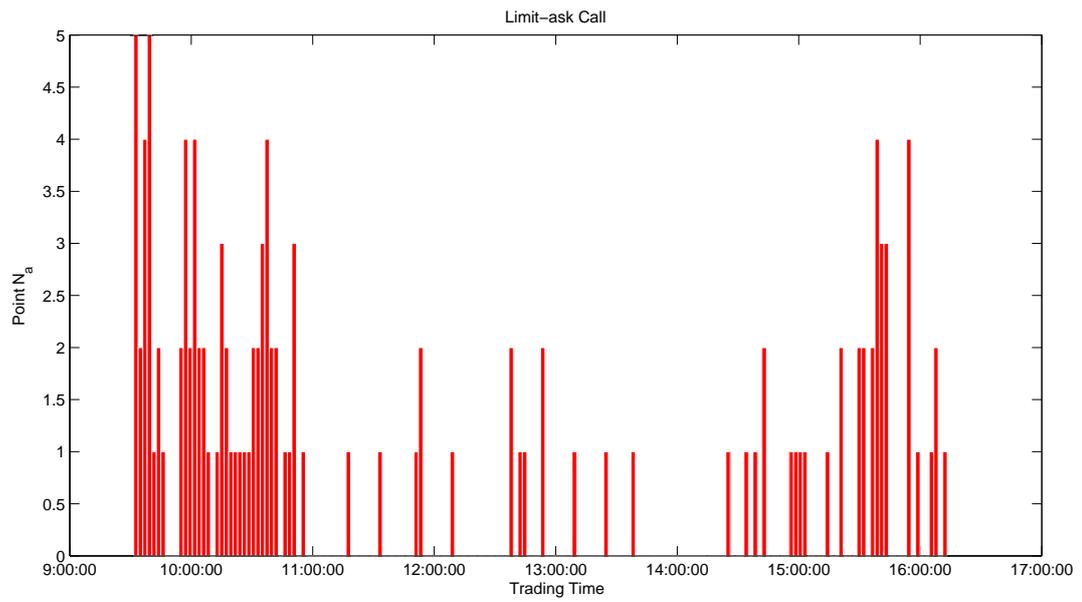
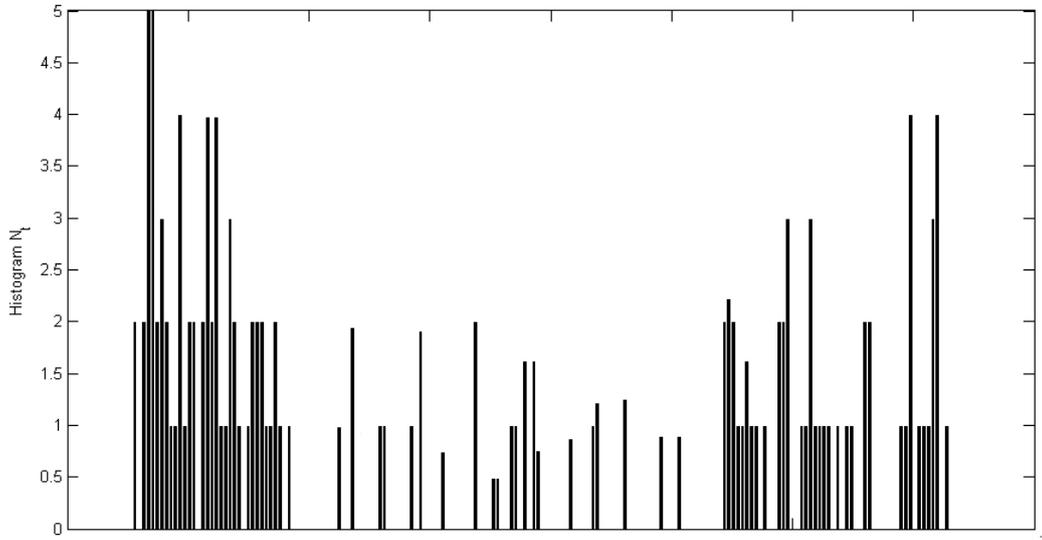
Note: This table shows the estimated results of branching ratios of three days around the positive jump day: October 257, 2011, for the call options data by using bivariate Hawkes processes.  $\theta_{11}$  and  $\theta_{22}$  represents for the branching ratio of the self-excitation effect,  $\theta_{12}$  represents for the branching ratio of the mutual-excitation effect that event 2 to event 1, and  $\theta_{21}$  represents for the mutual-excitation effect that event 1 to event 2. In this study, event 1 represents for the future orders, event 2 represents for the option orders. A represents for the total ask orders,  $A_L$  represents for the limit-ask orders, and  $A_M$  represents for the MC-ask orders, B represents for the total bid orders,  $B_L$  represents for the limit-bid orders, and  $B_M$  represents for the MC-bid orders.

**Table 12: Branching Ratio for Put Option of Three Days around the Positive Jump: 2011.10.27 of the Bivariate Hawkes Process**

		ATM						ITM						OTM					
		B		T		A		B		T		A		B		T		A	
		$\theta_{11}$ $\theta_{21}$	$\theta_{12}$ $\theta_{22}$																
$K_1$	A	0.8084	0.0188	0.7641	0.0000	0.1908	0.0213	0.7123	0.2311	0.7577	0.0593	0.1919	0.0025	0.8452	0.0153	0.7640	0.0004	0.1911	0.0139
	$A_L$	0.0000	0.5922	0.0000	0.5116	0.0170	0.7466	0.0943	0.5908	0.0000	0.5877	0.0213	0.5450	0.0209	0.5468	0.0000	0.5872	0.0000	0.6864
	$A_M$	0.8531	0.0000	0.7664	0.0000	0.1919	0.0000	0.8533	0.0000	0.7507	0.0000	0.1920	0.0000	0.8512	0.0003	0.7643	0.0000	0.1868	0.0118
	B	0.0000	0.8048	0.0000	0.7090	0.0000	0.7466	0.0000	0.6259	0.0002	0.5335	0.0000	0.7925	0.0000	0.7536	0.0000	0.7324	0.0002	0.7727
	$B_L$	0.8313	0.0000	0.7652	0.0000	0.1922	0.0000	0.8523	0.0000	0.7644	0.0000	0.1922	0.0000	0.8529	0.0000	0.7643	0.0000	0.1937	0.0002
	$B_M$	0.0000	0.8023	0.0000	0.7708	0.0000	0.7264	0.0131	0.6868	0.0000	0.7730	0.0000	0.7299	0.0033	0.7298	0.0000	0.6702	0.0167	0.7140
		0.8530	0.0000	0.7597	0.0696	0.1910	0.0039	0.8372	0.0350	0.7602	0.0358	0.1923	0.0002	0.8522	0.0692	0.7603	0.2171	0.1920	0.0000
		0.0000	0.5739	0.0000	0.4240	0.0517	0.4458	0.0457	0.5357	0.0000	0.5150	0.0330	0.5138	0.0053	0.5182	0.0000	0.3780	0.0000	0.5401
$K_2$	A	0.8609	0.0122	0.7774	0.0035	0.1928	0.0000	0.8528	0.0000	0.7350	0.0020	0.1919	0.0000	0.8525	0.0000	0.7611	0.0000	0.1913	0.0003
	$A_L$	0.0000	0.8456	0.0002	0.7489	0.0001	0.6913	0.0115	0.7344	0.0010	0.8463	0.0000	0.6939	0.0024	0.7320	0.0000	0.7397	0.0000	0.7276
	$A_M$	0.8532	0.0000	0.7645	0.0000	0.1934	0.0000	0.8554	0.0000	0.7643	0.0000	0.1912	0.0002	0.8531	0.0000	0.9272	0.0101	0.1920	0.0000
	B	0.0000	0.7970	0.0000	0.6976	0.0000	0.7668	0.0000	0.6617	0.0000	0.4897	0.0000	0.7743	0.0000	0.7678	0.0028	0.6383	0.0000	0.8081
	$B_L$	0.8504	0.0368	0.6947	0.0256	0.1921	0.0000	0.8462	0.0483	0.7155	0.0621	0.1920	0.0000	0.8445	0.1327	0.6939	0.2499	0.1927	0.0240
	$B_M$	0.0233	0.5966	0.0000	0.5675	0.0035	0.6556	0.0532	0.4740	0.0006	0.6100	0.0000	0.6129	0.0174	0.5800	0.0409	0.7485	0.0063	0.5074
		0.8529	0.0000	0.7643	0.0000	0.1902	0.0213	0.8585	0.0000	0.7645	0.0000	0.1917	0.0000	0.8450	0.0000	0.7642	0.0000	0.1923	0.0000
		0.0000	0.8344	0.0000	0.7599	0.0000	0.8121	0.0183	0.6393	0.0015	0.5748	0.0000	0.7738	0.0000	0.7116	0.0000	0.7667	0.0000	0.7169
$K_3$	A	0.8637	0.0096	0.7640	0.0000	0.1917	0.0000	0.8524	0.0000	0.7633	0.0000	0.1920	0.0000	0.8532	0.0000	0.7638	0.0000	0.1889	0.0024
	$A_L$	0.0082	0.7759	0.0000	0.8426	0.0047	0.7716	0.0159	0.5875	0.0000	0.7668	0.0000	0.7519	0.0000	0.7108	0.0000	0.8237	0.0000	0.7124
	$A_M$	0.8530	0.0000	0.7605	0.1385	0.1918	0.0000	0.8425	0.0735	0.7579	0.0839	0.1918	0.0000	0.8530	0.0000	0.7627	0.2216	0.1918	0.0000
	B	0.0238	0.5750	0.0000	0.7898	0.0030	0.6946	0.0386	0.5607	0.0000	0.5439	0.0000	0.5314	0.0254	0.5502	0.0000	0.4991	0.0025	0.5513
	$B_L$	0.8543	0.0172	0.7612	0.0000	0.1918	0.0000	0.8530	0.0000	0.7717	0.0000	0.1919	0.0000	0.8531	0.0000	0.7871	0.0020	0.1964	0.0003
	$B_M$	0.0000	0.5098	0.0000	0.8717	0.0023	0.7844	0.0088	0.6682	0.0000	0.7793	0.0000	0.7349	0.0000	0.6673	0.0001	0.7609	0.0002	0.7288
		0.8421	0.0056	0.7630	0.0000	0.1917	0.0000	0.8552	0.0000	0.7644	0.0000	0.1918	0.0000	0.8677	0.0118	0.8253	0.0537	0.1907	0.0000
		0.0004	0.7886	0.0006	0.8421	0.0000	0.8451	0.00005	0.6377	0.0000	0.5895	0.0000	0.7323	0.0000	0.7333	0.0006	0.6401	0.0000	0.8302
$K_4$	A	0.8535	0.0394	0.7614	0.0972	0.1937	0.0384	0.8516	0.0051	0.7643	0.0000	0.1919	0.0000	0.8531	0.0000	0.7642	0.0013	0.1869	0.0510
	$A_L$	0.0299	0.5714	0.0000	0.5642	0.0000	0.5765	0.0471	0.5973	0.0000	0.9996	0.0221	0.4145	0.0027	0.5633	0.0000	0.8050	0.0000	0.7439
	$A_M$	0.8482	0.0000	0.7642	0.0000	0.1925	0.0000	0.8529	0.0000	0.7641	0.0000	0.1917	0.0000	0.8526	0.0000	0.7603	0.0000	0.1915	0.0000
	B	0.0016	0.8011	0.0000	0.5849	0.0000	0.7845	0.0000	0.6834	0.0000	0.9691	0.0000	0.6972	0.0000	0.7596	0.0000	0.8487	0.0000	0.9225
	$B_L$	0.8534	0.0001	0.7644	0.0000	0.1919	0.0000	0.8528	0.0000	0.7640	0.0000	0.1916	0.0000	0.8531	0.0000	0.7647	0.0000	0.1917	0.0000
	$B_M$	0.0076	0.7924	0.0000	0.7822	0.0000	0.6972	0.0206	0.6299	0.0000	0.8616	0.0000	0.7010	0.0000	0.7205	0.0000	0.8540	0.0000	0.7832
		0.8464	0.0615	0.7629	0.0218	0.2005	0.0546	0.8531	0.0000	0.7643	0.0000	0.1916	0.0000	0.8481	0.1024	0.7580	0.0391	0.1931	0.0024
		0.0279	0.6391	0.0000	0.4720	0.0000	0.7346	0.0473	0.5921	0.0000	0.9976	0.0228	0.3871	0.0000	0.6006	0.0025	0.6403	0.0000	0.6206
$K_5$	A	0.8518	0.0000	0.7568	0.0000	0.1888	0.0001	0.8528	0.0000	0.7637	0.0000	0.1919	0.0000	0.8426	0.0010	0.7640	0.0000	0.1916	0.0000
	$A_L$	0.0041	0.6383	0.0000	0.7378	0.0000	0.8303	0.0175	0.6326	0.0000	0.8555	0.0000	0.6758	0.0009	0.7389	0.0000	0.7714	0.0000	0.7905
	$A_M$	0.8530	0.0000	0.8220	0.0437	0.1919	0.0000	0.8531	0.0000	0.7636	0.0000	0.1923	0.0000	0.8530	0.0000	0.7641	0.0000	0.1910	0.0000
	B	0.0000	0.7367	0.0012	0.7255	0.0000	0.8407	0.0000	0.6647	0.0000	0.9613	0.0000	0.6853	0.0000	0.7713	0.0000	0.7618	0.0000	0.7963
	$B_L$	0.8518	0.0000	0.7568	0.0000	0.1888	0.0001	0.8528	0.0000	0.7637	0.0000	0.1919	0.0000	0.8426	0.0010	0.7640	0.0000	0.1916	0.0000
	$B_M$	0.0041	0.6383	0.0000	0.7378	0.0000	0.8303	0.0175	0.6326	0.0000	0.8555	0.0000	0.6758	0.0009	0.7389	0.0000	0.7714	0.0000	0.7905
		0.8530	0.0000	0.8220	0.0437	0.1919	0.0000	0.8531	0.0000	0.7636	0.0000	0.1923	0.0000	0.8530	0.0000	0.7641	0.0000	0.1910	0.0000
		0.0000	0.7367	0.0012	0.7255	0.0000	0.8407	0.0000	0.6647	0.0000	0.9613	0.0000	0.6853	0.0000	0.7713	0.0000	0.7618	0.0000	0.7963
$K_6$	A	0.8533	0.0000	0.7644	0.0000	0.1920	0.0170	0.8493	0.0232	0.7626	0.0275	0.1904	0.0000	0.8031	0.4048	0.7519	0.0246	0.1925	0.0000
	$A_L$	0.0131	0.6169	0.0000	0.6103	0.0020	0.6319	0.0351	0.5781	0.0191	0.4067	0.0000	0.5057	0.0163	0.7293	0.0006	0.7900	0.0100	0.5176
	$A_M$	0.8530	0.0000	0.7644	0.0000	0.1918	0.0000	0.8532	0.0000	0.7642	0.0000	0.1919	0.0000	0.8531	0.0000	0.7217	0.0000	0.1918	0.0000
	B	0.0000	0.8105	0.0000	0.6280	0.0004	0.8309	0.0000	0.6450	0.0000	0.5391	0.0000	0.9252	0.0000	0.7812	0.0004	0.8114	0.0000	0.7815
	$B_L$	0.7841	0.1595	0.7653	0.0000	0.1912	0.0000	0.8532	0.0000	0.7668	0.0124	0.1898	0.0000	0.8650	0.0000	0.7824	0.0019	0.1914	0.0000
	$B_M$	0.0221	0.6420	0.0000	0.7895	0.0047	0.7425	0.0094	0.6128	0.0029	0.7822	0.0000	0.7637	0.0143	0.8040	0.0006	0.8592	0.0000	0.7794
		0.8523	0.0000	0.7644	0.0000	0.1961	0.0831	0.8519	0.0020	0.7643	0.0001	0.1938	0.0000	0.7266	0.2693	0.7612	0.0253	0.1918	0.0000
		0.0350	0.6276	0.0000	0.5048	0.0098	0.7068	0.0296	0.5566	0.0117	0.3449	0.0000	0.5124	0.0338	0.7196	0.0006	0.4576	0.0178	0.6004
$K_7$	A	0.8536	0.0000	0.7643	0.0000	0.1922	0.0000	0.8528	0.0000	0.7410	0.0000	0.1918	0.0000	0.8502	0.0002	0.7643	0.0000	0.1919	0.0000
	$A_L$	0.0000	0.7769	0.0000	0.7185	0.0000	0.7885	0.0095	0.5925	0.0000	0.7290	0.0000	0.7609	0.0134	0.8049	0.0000	0.7757	0.0000	

Figure 1: Histogram of the Orders

(a) The limit order of call options



(b) The future orders

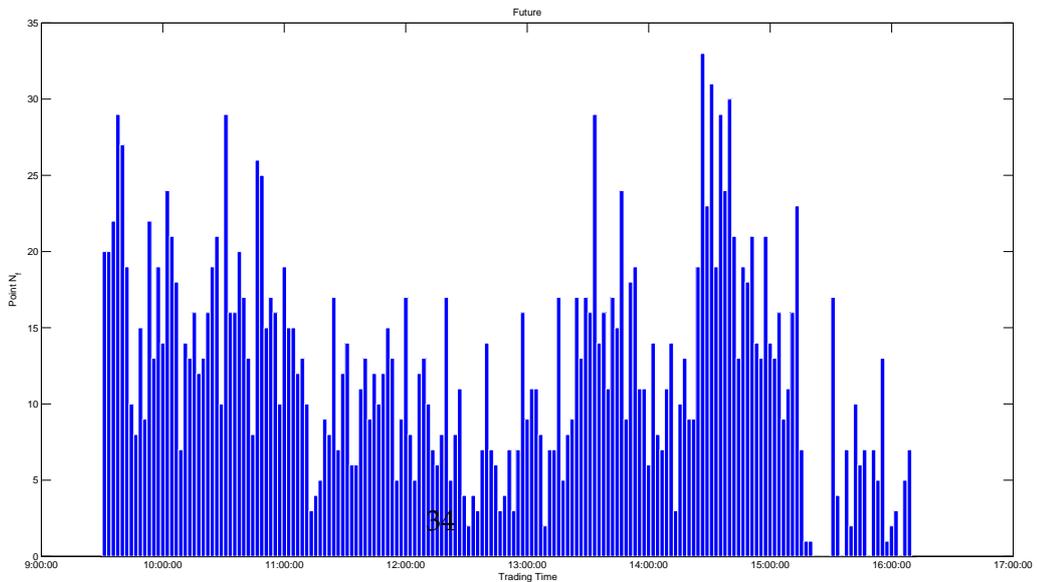
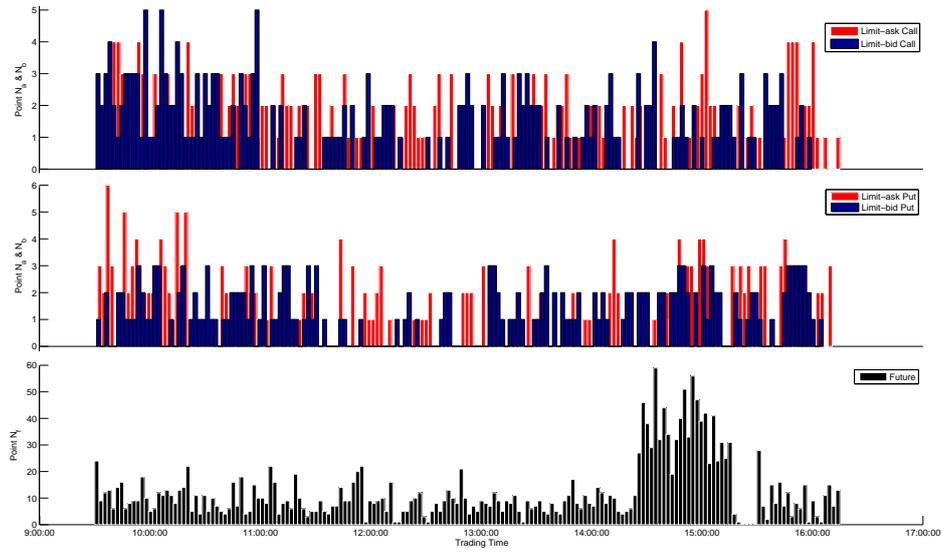
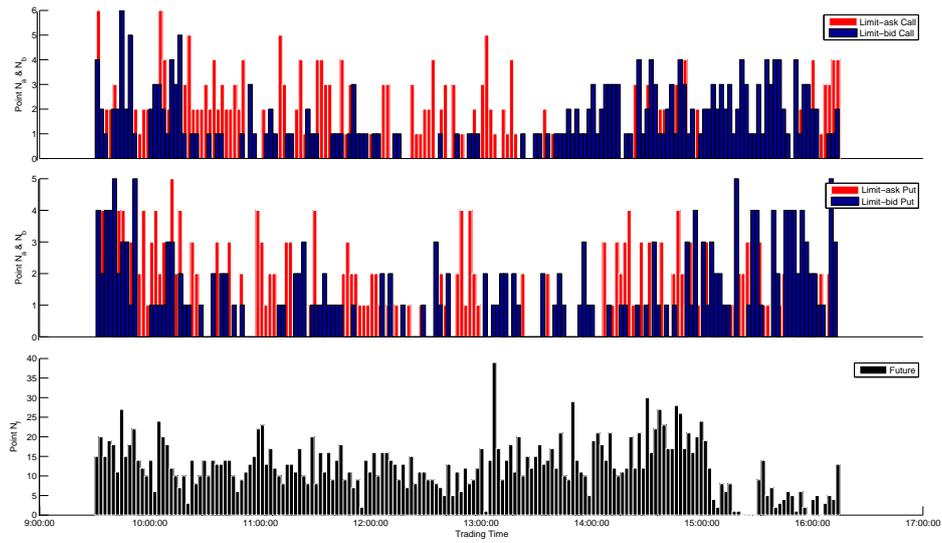


Figure 2: Histogram of the Futures and Option Orders

(a) Futures and limit ask and bid option orders for 5.24



(b) Futures and limit ask and bid option orders for 5.25



(c) Futures and limit ask and bid option orders for 5.26

